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Picard modular surface and families of automorphic forms

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Summary:

p-adic families of modular forms as constructed first by Hida and in greater generality by Coleman and their generalizations are a great tool in modern number theory

that appears in different parts of the Langlands program. In these talks I will try to explain the geometric construction of these families in the case of Picard automorphic forms. Geometric means that we will try to interpolate p-adically the sheaves on the Picard modular surface. As the local geometry of the surface depends on the behavior of the prime p in the quadratic field of the Shimura Datum, I will focus in the case where the prime is inert.

After studying the p-adic geometry of the Picard surface using the universal family of p-divisible groups that naturally live on it, we will study the overconvergence of a canonical filtration by analyzing the variation of the Hodge-Tate maps of these groups. Using this, we can p-adically interpolate the automorphic (coherent) sheaves on a strict neighborhood of the mu-ordinary locus, construct a compact p-adic operator acting on sections of these sheaves and use this datum to construct the Eignnvariety, a three dimensional p-adic variety that parametrizes p-adic congruences between overconvergent Picard automorphic forms.

If time permit, I will try to explain how we can use this object to construct classes in some Selmer groups as predicted by the Bloch-Kato conjecture using a method of Bellaiche and Chenevier.

Contents:

- The Picard modular surface, classical and modular description, primes of good reduction, Picard modular (automorphic) forms (classical and modular description).

- Families of p-divisible groups : definition, Lie-Algebra, examples, Dieudonne modules, Hodge-Tate map, Faltings crystals, and O-action.

- Back on the Picard surface: Hasse invariants, duality, p-adic geometry, stratification, Newton-Hodge polygons, overconvergent modular forms.

- Canonical filtration on the \mu-ordinary locus, degree, crystalline periods and results on the Hodge-Tate map.
Lifting the Kernel of Frobenius

- Constructing the Eigenvariety: explaining the idea of the eigenvariety machine, constructing the interpolation of automorphic sheaves and the Up operator, results on analytic continuation (classicity) and property of the Eigenvariety

- Bellaiche Chenevier's method: Rogawski's transfer, corresponding point on the eigenvariety, adaptation of Ribet's lattice argument, possible extensions and control of the ramification.