WORKING GROUP ON "p-ADIC L-FUNCTIONS FOR CM FIELDS"

The goal of this working group is to understand the construction of p-adic L-functions given in [Kat78]. Other references to be considered are [Ser73] and [DR80]. In [Ser73], the author introduced and study p-adic modular forms in order to construct p-adic *zeta* functions for totally real fields. In [DR80] the construction of *abelian* p-adic L-functions for totally real number fields is performed following the method initiated in [Ser73]. As was already observed in [Ser73, §5.6] to have a more complete theory was necessary to introduce p-adic Hilbert modular forms point which was exploited in [DR80]. The paper on which we are interested [Kat78] treats the construction of *abelian* p-adic L-functions for CM number fields using p-adic Hilbert modular forms too.

This working group is structured as above, where each reference is to [Kat78]. These lectures can be considered in two groups: in lectures 2 to 9 we introduce all the theory needed, then in lectures 10 and 11 we construct the p-adic L-functions and we prove the main properties.

Lecture 0 **Introduction and overview** by Daniel: 19/02, 10h-12h, UPC room 409 in Omega Building.

As a motivation we recall the structture of Serre's construction of the Kubota-Leopold p-adic zeta function. In the second part of this lecture we will try to explain the situation considered in [Kat78].

Lecture 1 **Hilbert modular forms** by Santiago: 13/03, 17:30-19:30, UB in IMUB room.

Corresponds to §1.0, §1.1, §1.2 and §1.3 where the goal is to introduce the theory of Hilbert modular forms. An interesting result proved by Ribet and used in §1.2.14 to provide the q-expansion principle, is the fact that the geometric fibres of $\mathcal{M}(\mathfrak{c}, \Gamma_{00}(N))$ over $\mathrm{Spec}(\mathbb{Z})$ are geometrically irreducible. This result was proved by Ribet and has an interesting historical importance: In december 1973, Deligne explained to Serre a program to construct abelian p-adic L-functions for totally real number fields using a p-adic theory of Hilbert modular forms. Moreover, Deligne pointed out that this p-adic theory would be possible if enough was known about the moduli space of HBAV (i.e. $\mathcal{M}(\mathfrak{c}, \Gamma_{00}(N))$ which was introduced by Rappoport). Finally, Ribet's result and Rapoport's thesis removed the obstacles to Deligne's program and then the paper [DR80] was written.

Lecture 2 **Complex Hilbert modular forms** by Francesc: 03/04, 15:00-17:00, UPC room 409 in Omega Building.

Corresponds to §1.4, §1.5, §1.6, §1.7 and 1.8. Some facts that seems useful to be keeped in mind are: the introduction in §1.8 of C^∞ $\mathfrak{c}$-Hilbert modular forms and the *Hodge short exact sequence canonically split.*

Lecture 3 **p-adic Hilbert modular forms** by Oscar: 10/04, 17:30-19:30, UB in IMUB room.

Corresponds to §1.9, §1.10, §1.11 and §1.12. §1.11. The goal is to explain the p-adic theory needed to perform the main construction of the work. It is interesting to remark, as mentionned in the introduction, the possibility of p-adic interpolation of the L-values considered in this work springs from the fact of a p-adic analogue for ordinary abelian varieties of the splitting explained in §1.8.

Lecture 4 **(Generalization of)Damerell's formula** by Daniel: 17/04, 15:00-17:00, UPC room 409 in Omega Building.

Corresponds to §2.1, §2.2, §2.3 and §2.4. The goal of these lectures is to explain the statement and the proof of Theorem 2.4.5 which is equivalent to a Shimura's generalization of a classical Damerell's formula.

Lecture 5 **p-adic Demerell's formula** by Xevi: 24/04, 17:30-19:30, UB in IMUB room.

Corresponds to §2.5 and §2.6. One goal of this lecture is to explain the statement of Theorem 2.6.7 where a p-adic version of the Demerell's formula is stated. The second goal is to state and prove theorem 2.6.36, this result gives a description of the formula in Theorem 2.6.7 in terms of theta operators (which are the Hilbert analogues of the classical $q\frac{d}{dq}$).

Lecture 6 **Eisenstein Series** by Marc: 08/05, 17:30-19:30, UB in IMUB room.

Corresponds to Chapter III. The goal of this lecture is to construct Eisenstein series (§3.2 and §3.3), construct p-adic Eisenstein series (§3.4) and apply results of lectures 5,6 and 7 to these Eisenstein Series (§3.5).

Lecture 7 **The Eisenstein Measure** by Francesca: 15/05, 15:00-17:00, UPC room 409 in Omega Building.

Corresponds to Chapter IV. In §4.0 and §4.1 some generalities about p-adic measures over profinite abelian groups are explained, both sections are independent of the rest of the paper. Using results from chapter III in §4.2 an example of such measures is given: the *Eisenstein Measure*. Moreover, q-expansion and functional equation of this measure are obtained.

Lecture 8 **Statement of the main theorem** by Daniel: 22/05, 17:30-19:30, UB in IMUB room.

Corresponds to §5.0, §5.1 and §5.2 where notations about Hecke characters and HBAV with complex multiplication are introduced and studied. Finally in §5.3 the main theorem of the paper is stated.

Lecture 9 **Proof of the main theorem** by Santiago: 29/05, 15:00-17:00, UPC room 409 in Omega Building.

Corresponds to §5.4, §5.5 and §5.6. In this lecture we finally perform the construction of the p-adic measure giving rise the p-adic L-function (§5.4), prove interpolation property (§5.5) and functional equation (§5.6). Here we mix and use all the theory and results developed along the working group.

References

[DR80] P. Deligne and K. Ribet. Values of abelian L-functions at negative integers over totally real fields. *Invent. Math.*, 59:227–286, 1980.

[Kat78] N. Katz. p-adic L-functions for CM fields. *Invent. Math.*, 49:199–297, 1978.

[Ser73] J.-P. Serre. Formes modulaires et fonctions zêta. In *Proc. 1972 Antwerp Summer School*, volume 350 of *Springer Lecture Notes in Math.*, pages 191–268. 1973.