

PIERRE ANGLÈS

Geometric algebras and spinors

(Dedicated to the memories of Pertti Lounesto, Jaime Keller and Artibano Micali)

In these days the angel of topology and the devil of abstract algebra fight for the soul of each individual mathematical domain.

Hermann Weyl, *Invariants*, 1939, *Gesammelte Werke*, Band III, page 681.

Summary

The lecture will be divided into three parts.

The first one is devoted to the structure of the Clifford algebra and will deal with all the main mathematical content of Clifford algebras.

After a short presentation of the history of Clifford algebras, we will analyze the concept of an algebra. As pointed out by Nicolas Bourbaki, W.K. Clifford introduced the algebras known as Clifford algebras and proved that they are tensor products of quaternion algebras or of quaternion algebras by a quadratic extension.

Elementary properties of quaternion algebras will be recalled. Basic definitions and properties of Clifford algebras will be given. The construction of a basis will be studied. Standard classical properties will be given. The covering groups for the standard orthogonal group, the special orthogonal group, the connected component of the identity of the orthogonal group of a regular quadratic finite dimensional space over \mathbb{R} or \mathbb{C} will be given. Clifford algebras for standard pseudo Euclidean spaces will be studied.

The second part deals with spinors in contemporary physics. We give the terminology and define Dirac Spinors, Weyl Spinors, Majorana Spinors, Weyl Majorana Spinors. The language of physicists concerning spin groups, spin bundles and spin connections will be studied. The links between spinors and the fundamental interactions will be analyzed.

The third part is devoted to a few comments on Spinors in Minkowski space-time. Spinorial coordinates, the Weyl Spinor space, dotted spinors, spinor tensors, Dirac spinors, Dirac Matrices, Chirality, Charge conjugation and Majorana Spinors are successively presented.

The conclusion will be a look to the algebraic foundation of the theory of twistors and a short analysis on the epistemological level of the importance of Geometric Algebras.