# PROJECTIVE TRANSFORMATIONS AS VERSORS

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ABSTRACT. Whenever I introduce people in machine vision and computer graphics to the wonders of conformal geometric algebra (CGA) to describe the Euclidean rigid body motions and similarities, they always ask: *'Can you also do projective transformations?'*. In those fields, projective transformations expressed in $4 \times 4$ homogeneous coordinate matrices are the standard, giving a nice integration of pinhole projective imaging and Euclidean motions. The naturally transforming conics are the usual primitives in many geometric modeling packages, and used to understand the real world data. CGA, with its spheres and circles, feels much too restrictive to them.

Our answer should be *'Yes, we can!'*, because we are of course convinced that geometric algebra can do all of geometry, elegantly, compactly, and advantageously. So, it is time to get specific, or we will lose credibility. Currently, there appear to be two proposals:

- Goldman [3] uses $\mathbb{R}^{4,4}$ to transcribe the $4 \times 4$ matrices into rotor form, following the general framework of [1]. He gives the bivector generators of all standard projective transformations, but has to employ some unusual constructions to incorporate projections. The 28 degrees of freedom in the rotors are not all used, and the geometrical meaning of the blades in this model is unclear.
- One can explore the accidental group isomorphism between the groups $SL(4)$ and $\mathrm{Spin}(3,3)$ to represent the homogeneous coordinate matrices. Klawitter [4] has recently shown how to convert a versor from $\mathbb{R}^{3,3}$ to a $4 \times 4$ matrix, and vice versa. His approach is coordinate based, and the bivector generators of standard projective transformations are unfortunately not made explicit. He points out that the blades are line complexes (and thus unfortunately not conics).

My presentation exposes the $\mathbb{R}^{3,3}$-bivectors of the canonical projective transformations, thus bringing the $\mathbb{R}^{3,3}$ model of Klawitter closer to the practical flavor exhibited Goldman's work on $\mathbb{R}^{4,4}$. While doing so, we develop some geometrical and contextual insights in the $\mathbb{R}^{3,3}$ model (the 'space of lines'), showing how the cross ratio and duality are represented [2]. We briefly treat the geometrical meaning of its blades, and how this affects modeling reality. Conics do appear briefly and strangely.

As I started to investigate $\mathbb{R}^{3,3}$, I was hoping that it might give us an 'oriented projective geometry' [5] in which we can compute consistently with directed lines, a capability that would be very useful in machine vision and computer graphics, but currently lacking. I will show why this unfortunately fails: the odd versors that would make reflections explicit actually do something else (and rather useless).

Summarizing, neither $\mathbb{R}^{4,4}$ nor $\mathbb{R}^{3,3}$ are quite what we would like to present to the practitioners. More work is needed, and soon!

## REFERENCES

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