

Fourier Transformations in Conformal Geometric Algebra

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Conformal geometric algebra is very popular in applications of geometric algebra [1]. In recent years research and application of Clifford Fourier transformations is also flourishing [2]. Therefore the natural question is asked for the appropriate form of Fourier transformations in conformal geometric algebra. It seems suitable in a first step to select the bivector square roots of minus one [3,4] in the kernel construction. Moreover the question is for the proper scalar function of the position and frequency vectors in an exponential kernel. We basically suggest to use the inner product of conformal points, and investigate the consequences of this choice. Interesting strong relations to linear canonical transformations [5], such as the Fresnel transformation are explained. Then several standard properties of the thus established conformal geometric algebra Fourier transform (confFT) are shown. We further intend to discuss eigenfunctions, carefully taking the non-commutativity of conformal object signal functions and the confFT kernel into account.

References

[1] E. Hitzer, T. Nitta, Y. Kuroe, *Applications of Clifford's Geometric Algebra*, Adv. Appl. Clifford Alg., Vol. 23, Online First, March 2013, (2013), pp. 377-404. DOI: 10.1007/s00006-013-0378-4. Preprint: <http://arxiv.org/abs/1305.5663>.

[2] E. Hitzer, S.J. Sangwine (eds.), "Quaternion and Clifford Fourier transforms and wavelets", Trends in Mathematics **27**, Birkhauser, Basel, 2013.

[3] E. Hitzer, J. Helmstetter, R. Ablamowicz, *Square roots of -1 in real Clifford algebras*, in E. Hitzer, S.J. Sangwine (eds.), "Quaternion and Clifford Fourier transforms and wavelets", Trends in Mathematics **27**, Birkhauser, Basel, 2013, pp. 123-153. DOI: 10.1007/978-3-0348-0603-9_7, Preprints: <http://arxiv.org/abs/1204.4576>, http://www.tntech.edu/files/math/reports/TR_2012_3.pdf First published in K. Guerlebeck (ed.), Proc. of The 9th Int. Conf. on Clifford Algebras and their Applications, (2011).

[4] E. Hitzer, *The quest for conformal geometric algebra Fourier transformations*, In T. Simos, G. Psihoyios and C. Tsitouras (eds.), Numerical Analysis and Applied Mathematics ICNAAM 2013, AIP Conf. Proc. 1558, pp. 30-33 (2013). DOI: 10.1063/1.4825413, Preprint: <http://vixra.org/abs/1310.0248>

[5] K. B. Wolf, *Integral Transforms in Science and Engineering*, Mathematical Concepts and Methods in Science and Engineering, Vol. 11, Springer, 1979, chapter 9. Online text: <http://www.fis.unam.mx/~bwolf/integral.html>