

Fq functions	
Function signature	Description
Zn(n)	If n is an integer >1, this function constructs the ring $\mathbb{Z}/(n)$ . If $A=\mathbb{Z}/(n)$ , and k is any integer, the expression $k>>A$ yields the value $k \bmod n$ .
[B,b]= extension (A, f(T), 'a') [B,b]=extension (A, [1,c1,...,cr], 'a') base(B) prime_field(F)	<p>If A is a ring, f(T) a monic polynomial with coefficients in A, and a is an identifier, then the first statement binds B and b to the quotient <math>A[T] / (f(T))</math> and to the class of T modulo f(T), which is named a (or x if 'a' is not supplied). The second signature is equivalent to the first when <math>f(T) = T^r + c_1T^{r-1} + \dots + c_r</math>. If A is a field and f(T) is irreducible over A, then B is a field of degree r over A.</p> <p>The ring A can be recovered from B with the command base(B). The elements 1, a, <math>a^2</math>, ..., <math>a^{r-1}</math> form a linear free basis of B over A which is called the <i>natural</i> or <i>standard basis</i>.</p> <p>The minimum subfield of a finite field F can be obtained with prime_field(F) (it is <math>\mathbb{Z}/(p)</math>, where p is the characteristic of F).</p>
K_(a)	If the domain of an object a is a ring (which may be a field), this function returns $A :: \text{Ring}$ (or $A :: \text{Field}$ ). Examples: $K_{(5)}$ returns $\mathbb{Z} :: \text{Ring}$ , $K_{(1/5)}$ returns $\mathbb{Q} :: \text{Field}$ .
md_mult(f,g,h) md_mul = md_mult	Delivers the remainder of the euclidean division of $f \cdot g$ by h.
md_power(f,n,h)	Delivers the remainder of the euclidean division of $f^n$ by h.
md_double_power(f,m,n,h)	Delivers the remainder of the euclidean division of $f^{m^n}$ by h.
period(a) order(a) period(f) exponent = period	For a non-zero element a of a finite field, the call period(a) delivers the minimum positive integer r such that $a^r = 1$ , which coincides with order(a). If f is a univariate polynomial with variable x with coefficients in a finite field, period(f) yields the minimum positive integer r such that $x^r = 1 \bmod f$ .
legendre (x, F) QR(F) QNR(F)	<p>If x is a non-zero element of the finite field F of characteristic <math>\neq 2</math>, legendre(x,F) returns 1 if x is quadratic residue in F and -1 otherwise. By convention, legendre(0) is 0. For <math>x \neq 0</math>, legendre(x,F) coincides with <math>(-1)^{(q-1)/2}</math>, where q is the cardinal of F.</p> <p>QR(F) yields the list of the non-zero quadratic residues of F. Similary, QNR(F) yields the list of the quadratic non-residues of F.</p>
is_irreducible(f,K) is_irreducible(f)	The first call tells us whether the polynomial f is irreducible over K. The second does the same, but taking $K=K_{(f)}$ .
get_irreducible_polynomial(K,r,symbol='X')	Produces a monic irreducible polynomial of degree r over K in the variable supplied by 'symbol', which is 'X' by default.
irr(q,r)	Yields the number of monic irreducible polynomials over $\mathbb{F}_q$ of degree r.
Tr(x,K,L) Tr(x,K) Tr(x)	Given an element x defined in an extension of K, it computes the trace of the conjugates of x with respect to a finite field extension L/K. if L is omitted, it takes L as the minimum extension where x is defined. If K is not defined, it takes K as the minimum field in the chain domain of x.
prime_field(F)	Given a domain F with a chain of fields $K_0 \subseteq K_1 \subseteq \dots \subseteq K_n = F$ , it returns the base field $K_0$ .
index_table(x)	Given an element x of order k, it return a table given by the pairs $[x^{**j},j]$

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GF(q,n,alpha) GF(q,n) GF(q,alpha) GF(q)	This function creates the finite field of $q^n$ elements with generative element represented by alpha. By default alpha = 'x'. If n is not given, the function creates the finite field of q elements. n can also be a polynomial or a list of coefficients and then the function creates the extension using that polynomial. q can also be a domain and then, the function creates an extension using a given polynomial.
order(k, n)	If $\gcd(k,n) = 1$ , the order of k in $\mathbb{Z}_n^*$ . Otherwise 'Error'
inverse(k,n)	If $\gcd(k,n) = 1$ , inverse(k,n) computes a positive integer k' such that $k' < n$ and $k'k \equiv 1 \pmod n$ . Otherwise, "Error"
mult(n,k,b) bpow(n,k,b) quot(n,k,b) power(n,k,m)	The value of these expressions is $n \cdot k \pmod{2^b}$ , $n^k \pmod{2^b}$ and $(m / k) \pmod{2^b}$ , respectively. In the latter case, k has to be odd. The function bpow(n,k,b) coincides with power(n,k,2^b), as power(n,k,m) computes $n^k \pmod m$ . These functions are used, for example, in the definition of the next two.
jacobi(a,n) legendre(a,n)	Computes the Jacobi symbol $(a/n)$ of two integers a and n, which must be odd and positive. The PyM implementation is based on Algorithm 2.3.5 of Crandall-Pomerance-2005. If n is prime, it coincides with the Legendre symbol $(a/n)$ , which is 0 if a is divisible by n and otherwise it is +1 or -1 according to whether a is or is not a quadratic residue mod n. It coincides with legendre(a,Z_n)
nroot(n,k,b) nsqroot(n,b) sqroot(a,F)	If n is an odd integer and k is either odd or 2, nroot(n,k,b) computes an integer $r < 2^b$ such that $r^k \equiv 1 \pmod{2^b}$ , which is a kth root of $n^{(-1) \pmod{2^b}}$ . The function nsqroot(n,b) is equivalent to nroot(n,2,b). These functions are used in the definition of is_perfect_power(n). sqroot(a,F) returns an element b in the domain F, such that $b^2 = a$ in the domain F.
cyclotomic_class(k, n, q) cyclotomic_class(k, n)	Assuming that q and n are positive integers and that $\gcd(q,n)=1$ , the call cyclotomic_class(k,n,q) supplies the q-cyclotomic class of k mod n, which by definition is the list $[k, q \cdot k, q^2 \cdot k, \dots, q^{r-1} \cdot k]$ , where the operations are done mod n and r is the least positive integer such that $q^r \cdot k \equiv 1$ .
cycloctomic_classes(n, q) cycloctomic_classes(n)	Assuming that q and n are positive integers and that $\gcd(q,n)=1$ , the function cycloctomic_classes(n,q) furnishes the list of all the q-cyclotomic classes mod n. Finally, cycloctomic_classes(n) is defined as cycloctomic_classes(n,2).
frobenius(K)	
conjugates(x,K) conjugates(x)	List of conjugates of an element x in L over subfield K. By default K is the minimum subfield of L.
nm(x,K) nm(x)	It returns the trace of the conjugates of x over K. By default K is $\mathbb{Z}_n(2)$
is_primitive(f)	It returns if f is a primitive element in its domain.
primitive_root(K)	It returns a primitive element of the finite field F.
md_trace(h,m,f)	It is a auxiliar function of equal degree splitting. It computes the trace of an element in $F_2$ : $h+h^{**2}+h^{**4}+h^{**8}+ \dots + h^{**(2^{**}(m-1))} \pmod f$