Departament de Matemàtiques

Taula rodona

La recerca en temps de la Covid-19 (passat, present i futur)

S. Xambó

DMAT-UPC/BSC

11/11/2020

Topics

PyM: Mathematical computation

Error-correcting codes

Intersection theory and enumerative geometry

Algebra

Geometric calculus and algorithmic learning

Biographical studies

Outlook

PyM: Mathematical computation

PyM: `X/PyM.html`, `X=https://web.mat.upc.edu/sebastia.xambo`

[1] (2017, with S. Molina & N. Sayols) —enumerative arithmetic geometry

[2] (2018, with N. Sayols) —*Butlletí SCM* (post-quantum cryptography)

[3] (2019.11) Talk on post-quantum cryptography (UCA, Jornadas doctorales)

[4] (2020.01) Talk in "Red GAS" —can classical computation certify quantum computers? (Urmila Mahadev, 2018, *Classical Verification of Quantum Computations* & *Classical homomorphic encryption for quantum cirucuits*)

Error-correcting codes

[5] (2019, with R. Farré & N. Sayols): "On PGZ decoding of alternant codes". *Computational and Applied Mathematics* **38**, article 17 (2019), 1-13.

[6] (2020, with N. Sayols): "Computer Algebra Tales on Goppa Codes and McEliece Cryptography". *Mathematics in Computer Science* **14**, 457–469 (2020).

ECC2: `X/CC2/CC2-Book.html` (interface to ecclets)

[7]: *Error-correcting codes—A computational primer* (2020)

(Extended second edition of *Block Error-Correcting codes—A computational primer*, Springer, Universitext, 2003).

New chapters: Code-based post-quantum cryptography, Quantum codes, Convolutional codes.

§1.4 Parameter bounds

- 1.27 *Griesmer's upper bound and Griesmer's function* [**py** | **nb**]
- 1.28 *Plotkin's upper bound* [**py** | **nb**]
- 1.29 *Elias' upper bound* [**py** | **nb**]
- 1.30 *Johnson's upper bound* [**py** | **nb**]
- 1.31 *Krawtchouk's polynomials* [**py** | **nb**]
- 1.32 *Linear programming upper bound* [**py** | **nb**]
- 1.33 *The entropy function and the asymptotic bounds* [**py** | **nb**]

ecclets

Chapter 2. Finite Fields

Hands-on approach

§2.1 $\mathbb{Z}_n$ and $\mathbb{F}_p$

- 2.01 *Examples of computations in rings* $\mathbb{Z}_n$ [**py** | **nb**]
- 2.2 *Euler's totient function* φ [**py** | **nb**]

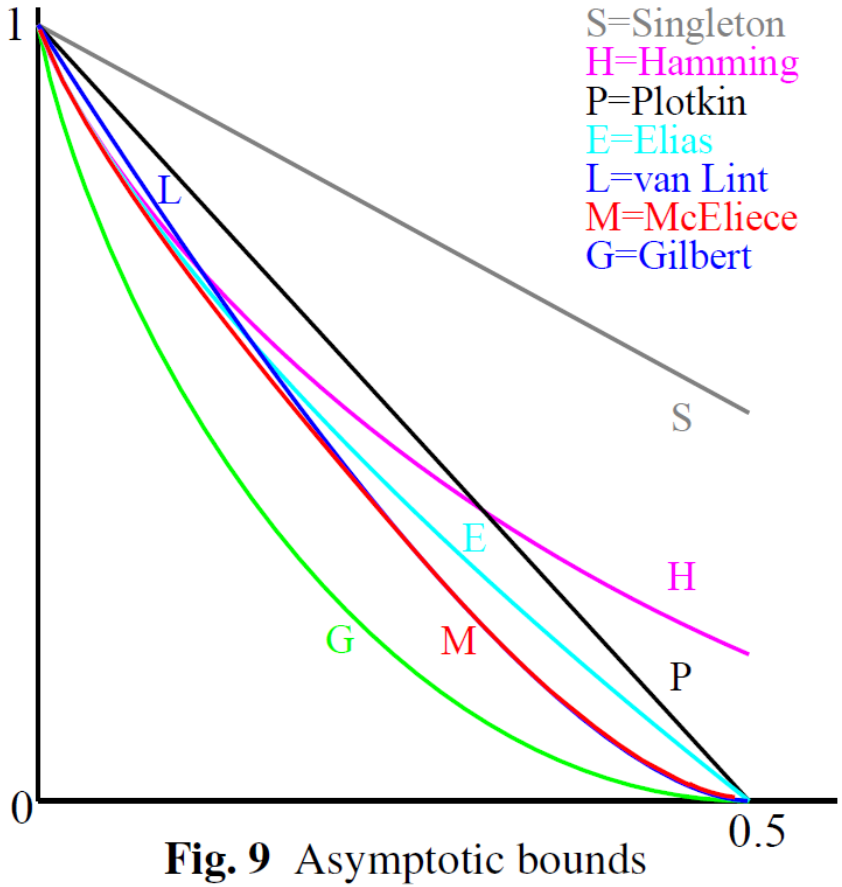


Fig. 9 Asymptotic bounds

```sage
## Linear Programming upper bound

from PyM import *

def LP(n,d,R = []):
    if odd(d):
        n = n + 1
        d = d + 1
    K = [krawtchouk(k,n) for k in range(1,n//2 + 1)]
    A = [[evaluate(k,0)] + [evaluate(k,j)
        for j in range(d,n+1,2)] for k in K]
    A = -matrix(A)
    alpha = symbol('a',n)
    alpha = [1] + alpha[d - 1:n:2]
    alpha = vector(alpha,'c')
    RT = A*alpha
    RT = [r for r in RT] + R
    cost = sum(alpha)
    m1,m2 = simplex(cost,RT)
    b = floor(m1)
    if odd(b):
        beta = stack([1-1/(b>>Q_)],alpha[1:])
        RT = A*beta
        RT = [r for r in RT] + R
        m1,m2 = simplex(cost,RT)
    return floor(m1),m2
```

ecclet 1.32

Intersection theory and enumerative geometry (ITEG)

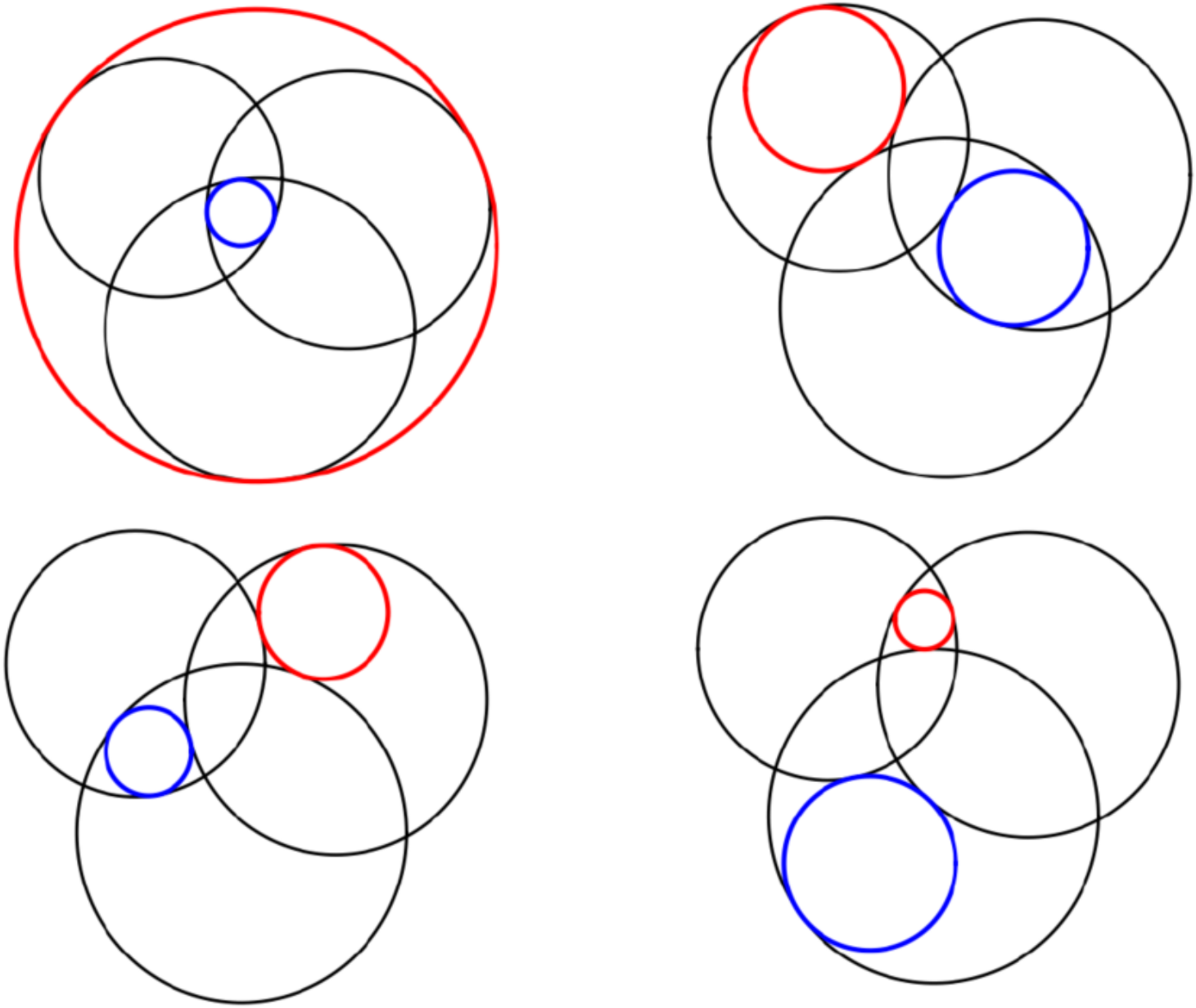
PyWIT: `X/PyWIT.html` (interface to witlets)

[8] *WIT: A symbolic system for computations in IT and EG (programmed in pure Python)*. Mini-course at IMUVA.

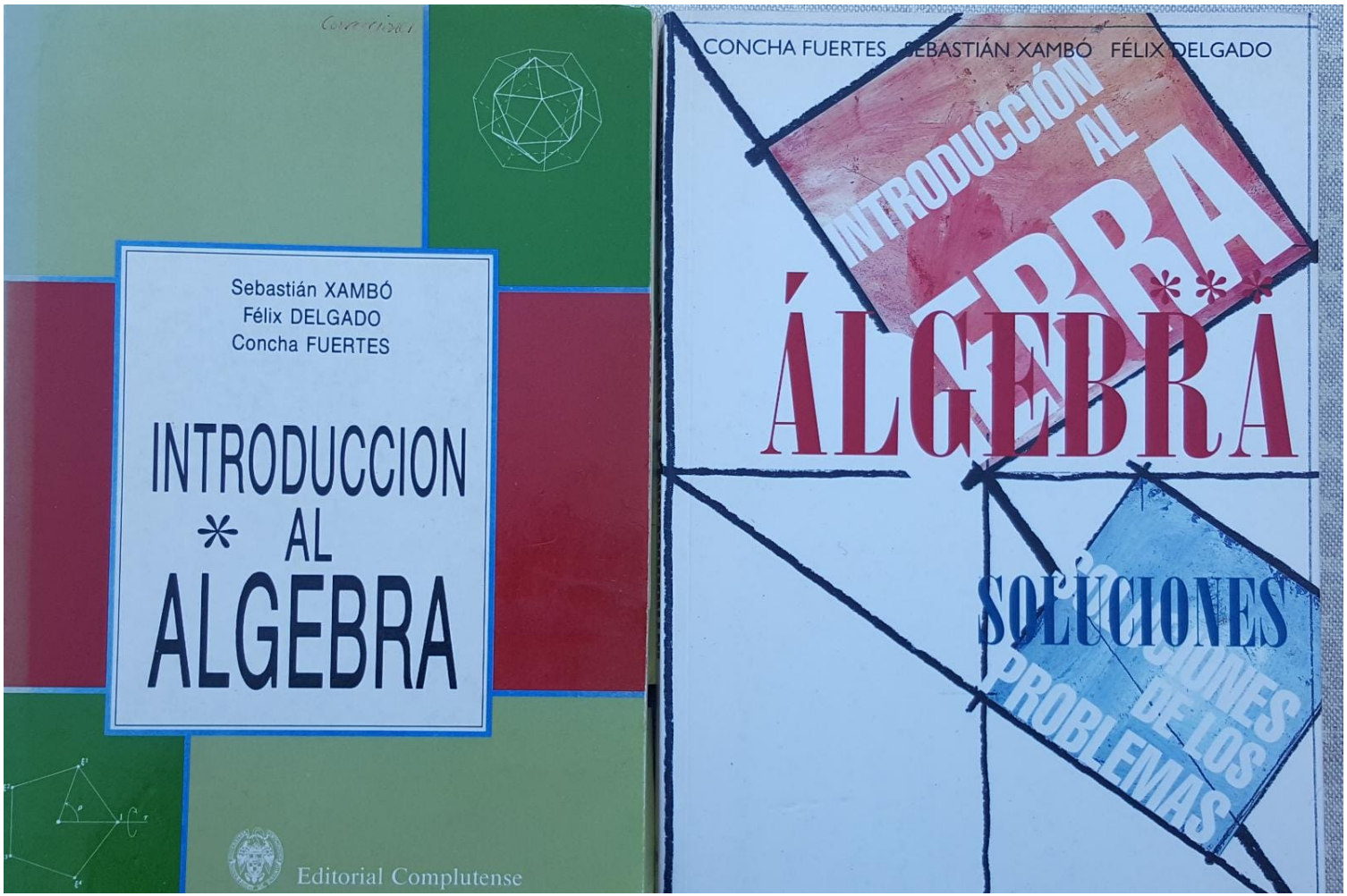
[9]: *Intersection Theory and Enumerative Geometry—A Computational Primer* (Springer, 2021)

Extended second edition of *Using intersection theory*, SMM (1996), 122 p.

New chapters: varieties of concurrent lines, singular plane cubics, twisted cubics, physics-driven enumerative geometry, enumerative geometry on moduli spaces.



Algebra



Second edition in a single volume. 420 p.

Geometric Calculus and Algorithmic learning

[10] (2019, *A Light Dream*. Homage to Eusebio Corbacho and Elena Martín Peinador.

[11] Organizer minisymposium at ICIAM-2019 (MS): *Systems, patterns and data engineering with geometric calculi*. Editor of a proceedings volume with the MS title, to appear in the ICIAM2019 SEMA SIMAI Springer Series, with A. Delshams as Editor-in-Chief.

[12] MS keynote lecture: *Geometric calculus techniques in science and engineering*.

[13] (MS, with E. U. Moya): *Geometric bio-inspired deep learning*. To appear in proceedings volume.

[14] (MS, with E. U. Moya): *Geometric calculus meets deep learning*. To appear in proceedings volume with the title *Geometric calculi for deep learning*.

[15] *Geometric Algebra. Mathematical Structures and Applications*. Mini-course at UNED (3-4 Oct 2019), Madrid.

[16] (with E. U. Moya, A. Sánchez, S. Salazar, J. Ortega, U. Cortés): "A bio-inspired quaternion local phase CNN layer with contrast invariance and linear sensitivity to rotation angles." *Pattern Recognition Letters* **131** (2020), 56-62.

[17]: *Theoretical Resources for Deep Learning*. Keynote lecture 2020.8.04 at the ICCA12 mini-symposium on Geometric Calculi and Deep Learning (organized with I. Zaplana)

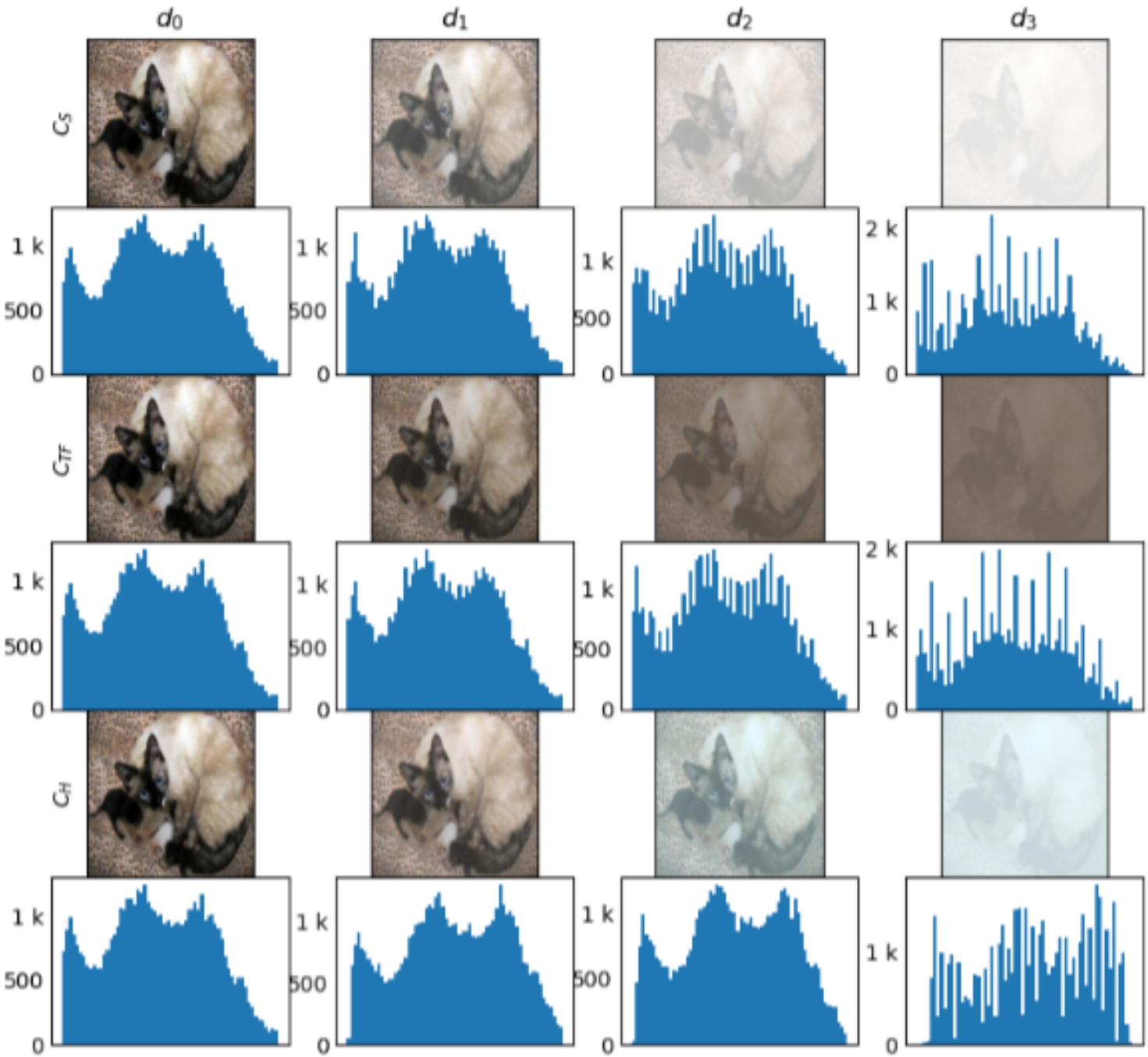
[18] (with E. U. Moya, A. Sánchez, S. Salazar, U. Cortés): "Trainable bio-inspired monogenic CNN layer for contrast invariance" (2020). Submitted to *IEEE Transactions on Neural Networks and Learning Systems*, Special Issue on Biologically learned/inspired methods for sensing, control and decision making.

[19] (with Joan Bruna): *Aprenentatge algorísmic i xarxes neuronals profundes* (54 p.). Submitted to the Butlletí de la SCM.

[20]: (with PhD student M. A. Gutiérrez): Research Plan in AI, UPC. Submitted 2020-11-02.

[21] (with N. Sayols) *Learning curves from a cloud of noisy points, with applications to bio-medical video images* (in preparation)

A Light Dream


d_0
d_1
d_2
d_3
C_S
C_{TF}
C_H
1 k
500
0
2 k

Biographical studies

Alessio Figalli: Magic, Method, Mission

Sebastià Xambó-Descamps (Universitat Politècnica de Catalunya (UPC), Barcelona, Catalonia, Spain)

This paper is based on [49], which chronicled for the Catalan mathematical community the Doctorate Honoris Causa conferred to Alessio Figalli by the UPC on 22nd November 2019, and also on [48], which focussed on the aspects of Figalli's scientific biography that seemed more appropriate for a society of applied mathematicians. Part of the Catalan notes were adapted to Spanish in [10]. It is a pleasure to acknowledge with gratitude the courtesy of the Societat Catalana de Matemàtiques (SCM), the Sociedad Española de Matemática Aplicada (SEMA) and the Real Sociedad Matemática Española (RSME) their permission to freely draw from those pieces for assembling this paper.

Origins, childhood, youth, plenitude

Alessio Figalli was born in Rome on 2 April 1984. His father Gennaro Figalli, now retired, was a professor of engineering, and his mother, Giuseppina Carola, is a teacher of Latin and Greek at a classical high school (*liceo classico*) in Rome. From Alessio's early years, in [45] Kevin Hartnett selects the following impressions:

> *As a kid Figalli liked to play soccer, watch cartoons, and hang out with his friends – and, he recalls, he always made the rational decision to get his homework done first, so that he could fully enjoy himself. "For me it was always a balance between how good a grade I could get and how much time I had to spend to get such a grade," he said. "I was always an optimizer, I wanted the best for the least effort."*

In the interview [45], conducted by Helga Rietz, Figalli talks of his first experiences in relation to mathematics:

> *As a child I liked math because it was easy for me. I was thinking of being an engineer. Then I had to decide, at age 13, which high school I would like to attend. There are many types in Italy, but the main ones that prepare for the university are scientific and classical lyceums. In the latter you learn Greek and Latin, philosophy, etc., and I chose this option. In Italy there is always the idea that the classical high school offers the broadest education and that with this training you can later study whatever you want…*

In his third year at high school, Antonio Corbo, a mathematician who worked at the same university as his father, suggested that Alessio participate in the Mathematical Olympiad. This led him to realise that there were mathematical problems whose solutions required inventiveness, and his aptitude for solving them, as well as the joy such magic insights brought him, were a truly revelatory experience. In the aforementioned interview [45], Figalli describes this experience:

> *At the Mathematical Olympiad, I met other teens who loved math. All of them dreamed of studying at the Scuola Normale Superiore in Pisa (SNSP), which offers a high level of education. Those who get one of the coveted scholarships do not have to pay anything. Living, eating and studying are free. I also wanted that.*
>
> *I concentrated on mathematics and physics on my own and managed to pass the entrance exam. The first year at Scuola Normale (SN) was tough. I didn't even know how to calculate a derivative, while my colleagues were much more advanced than me, since they came from the scientific lyceum.*

He undertook a methodical study plan that allowed him to catch up with his peers within a year. How far he went is revealed by the following episode. At the beginning of his second year, he began to read a highly technical work that Luigi Ambrosio and Xavier Cabré had recently written [4]. Ambrosio expected that the novice student would have to persist for quite some time to make some progress. But the surprise for Ambrosio was, as stated in [42], that "Alessio came to see me less than a week later and I realised that he understood everything". This step marked Figalli's course in mathematical research: one year later he completed the bachelor's degree [22] and in the following three years he obtained the master's degree [23] and the doctorate [25], always with full honours. His doctoral thesis was supervised by Luigi Ambrosio and Cédric Villani (Fields Medal 2010). In the words of David Jerison (MIT), "Alessio is incredibly fast. Quick on the essentials and quick to isolate the important points".

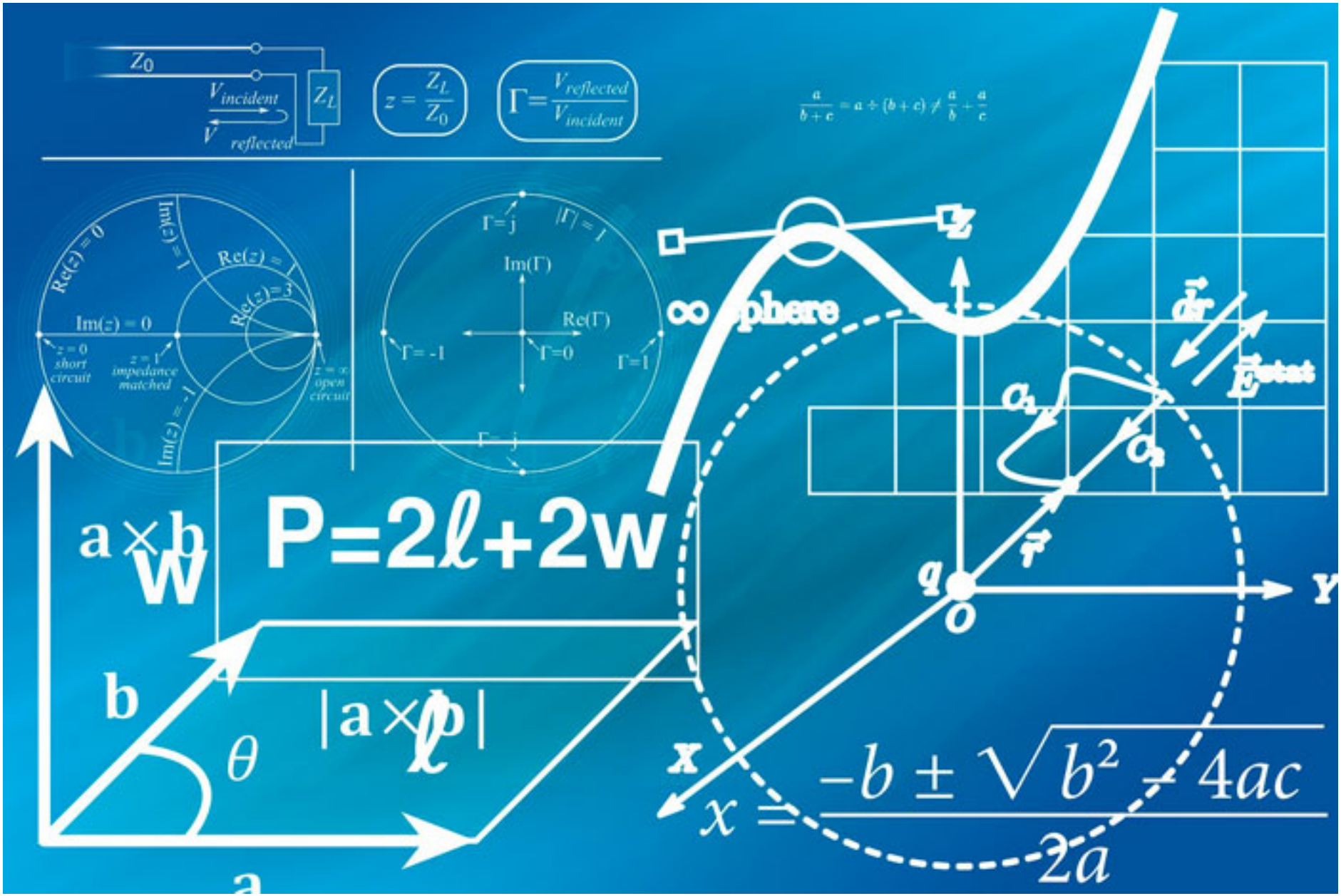
Figalli's scientific output after his doctorate is surprising in all respects. Its leading role in the global mathematical landscape has steadily increased over the years. From the point of view of his academic positions, three phases can be distinguished. The first comprises of two terms: 2007–08, as a researcher at the University of Nice, and 2008–09, as *Professeur Hadamard* at the *École Polytéchnique de Palaiseau*. The second phase consists of the seven terms at the *University of Texas at Austin* (from 2009–10 to 2015–16): the first two as an associate professor (and *Harrington Faculty Fellow* in the first) and as professor in the following five (*R. L. Moore Chair* in the last three). Finally, since October 2016 he has been a professor at the ETH (Eidgenössische Technische Hochschule Zürich) and director of the Forschungsinstitut für Mathematik (FIM) since September 2019.

[22]: "Alessio Figalli: Magic, Method, Mission". *EMS Newsletter* **9**, 117 (2020), 15-25.

X. Cabré, J. Serra, X. Ros-Oton, X. Fernández-Real

[23] (2019): M. Atiyah, *in memoriam*. *SCM/Notícies*.

[24] (2019.06): "Lluís A. Santaló Sors: Semblanza de un investigador" (RSME-UIMP)


Z_0
$V_{incident}$
$V_{reflected}$
Z_L
$z = \frac{Z_L}{Z_0}$
$\Gamma = \frac{V_{reflected}}{V_{incident}}$
$\frac{a}{b+c} = a \div (b+c) \neq \frac{a}{b} + \frac{a}{c}$
Re(z) = 0
Im(z) = 1
Re(z) = 1
Re(z)=3
Im(z) = 0
z = 0 short circuit
z = 1 impedance matched
z = ∞ open circuit
Im(z) = -1
Γ=j
|Γ| = 1
Im(Γ)
Re(Γ)
Γ= -1
Γ=0
Γ=1
∞ sphere
a×b
W
P=2ℓ+2W
b
θ
|a×b|
ℓ
a
q
O
Y
x
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

References I

[1] S. Molina, N. Sayols, and S. Xambó-Descamps, "A bootstrap for the number of $\mathbb{F}_{q^r}$-rational points on a curve over $\mathbb{F}_q$," 2017.
https://arxiv.org/pdf/1704.04661.pdf.

[2] N. Sayols Baixeras and S. Xambó i Descamps, "Codis correctors d'errors i criptografia postquàntica," *Butlletí de la Societat Catalana de Matemàtiques*, vol. 33, no. 2, pp. 147–171, 2018.

[3] S. Xambó-Descamps, "Post-quantum cryptography," 2019.11.
https://web.mat.upc.edu/sebastia.xambo/PQ/s-cadiz.pdf.

[4] S. Xambó-Descamps, "Dos factas de $\mathbb{P}^1_K$ y sus ramificaciones," 2020.01.
https://web.mat.upc.edu/sebastia.xambo/PQ/s-jmmp.pdf.

[5] R. Farré, N. Sayols, and S. Xambó-Descamps, "On PGZ decoding of alternant codes," *Computational and Applied Mathematics*, vol. 38, no. 1, pp. 1–13, 2019.

[6] N. Sayols and S. Xambó-Descamps, "Computer Algebra Tales on Goppa Codes and McEliece Cryptography," *Mathematics in Computer Science*, vol. 14, pp. 457–469, 2020.

[7] N. Sayols and S. Xambó-Descamps, "error-correcting codes: a computational primer.," 2020.
Extended second edition of [25], https://web.mat.upc.edu/sebastia.xambo/CC2/CC2-Book.html.

[8] J. M. Miret, N. Sayols, and S. Xambó-Descamps, "WIT: A symbolic system for computations in IT and EG (programmed in pure Python)," 2020.01.
https://web.mat.upc.edu/sebastia.xambo/ITEG/s-wit-imuva-I.pdf,
https://web.mat.upc.edu/sebastia.xambo/ITEG/s-wit-imuva-II.pdf.

[9] J. M. Miret, N. Sayols, and S. Xambó-Descamps, "Intersection Theory and Enumerative Geometry—A Computational Primer," 2021.
extended edition of [26], https://web.mat.upc.edu/sebastia.xambo/WIT/UIT.html.

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[10] S. Xambó-Descamps, "A Light Dream," 2019.06.
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[11] S. Xambó-Descamps, "Systems, patterns and data engineering with geometric calculi," 2019.07.
https://web.mat.upc.edu/sebastia.xambo/ICIAM2019/GC&DL.html.

[12] S. Xambó-Descamps, "Geometric Calculus Techniques in Science and Engineering," 2019.07.
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[13] S. Xambó-Descamps, "Geometric bio-inspired Deep Learning," 2019.07.
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[14] S. Xambó-Descamps, "Geometric calculus meets deep learning," 2019.07.
https://web.mat.upc.edu/sebastia.xambo/ICIAM2019/GC&DL-08.pdf.

[15] S. Xambó-Descamps, "Geometric Algebra. Mathematical Structures and Applications," 2019.10.
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[16] E. U. Moya-Sánchez, S. Xambó-Descamps, A. S. Pérez, S. Salazar-Colores, J. Martínez-Ortega, and U. Cortés, "A bio-inspired quaternion local phase cnn layer with contrast invariance and linear sensitivity to rotation angles," *Pattern Recognition Letters*, vol. 131, pp. 56–62, 2020.

[17] S. Xambó-Descamps, "Theoretical Resources for Deep Learning," 2020.
Lecture at the ICCA12 mini-symposium on Geometric Calculi and Deep Learning.

[18] E. U. Moya, S. Xambó-Descamps, A. Sánxhez, S. Salazar, and U. Cortés, "Trainable bio-inspired monogenic CNN layer forcontrast invariance," 2020.
Submitted to.

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Submitted to *Butlletí de la SCM*.

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Research Plan for PhD in AI.

[21] N. Sayols and S. Xambó-Descamps, "Learning curves from a cloud of noisy points, with applications to bio-medical video images," 2020.
In preparation.

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`https://web.mat.upc.edu/sebastia.xambo/Bios/Santalo/2019-Xambo--Santalo--Semblanza-de-un-investigador`.
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[26] S. Xambó-Descamps, *Using Intersection Theory*, vol. 7 of *Aportaciones Matemáticas, Nivel Avanzado*.
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122 p.