ICCA13

"To ML and beyond ..."

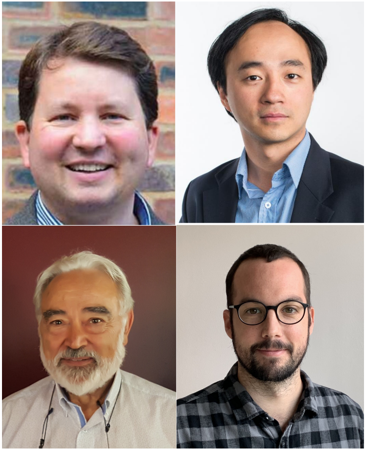
Unfolding and Contexts
Aims and Topics
Speakers

S. Xambó

IMTech & BSC

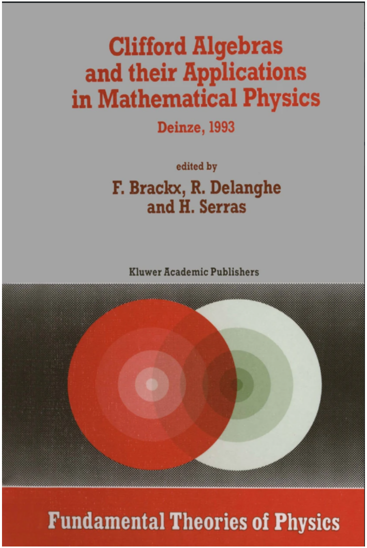
4-9/June/2023

Pierre P. DECHANT ▪ Yang-Hui HE



S. XAMBÓ-DESCAMPS ▪ Isiah ZAPLANA Interview.

International Conference on Clifford Algebras and Their Applications in Mathematical Physics



To machine learning and beyond: data science in mathematics, physics and engineering

- Data science concerns big data sets in high-dimensional spaces, along with considerations on the geometry and structure of these spaces, as well as efficient algorithms for high performance computations around
  - *transforming data*
  - *dimensionality reduction*
  - *data visualisation*
  - *optimisation*
  - *machine learning*
- Clifford geometric approaches are well suited to help addressing the intrinsically geometric aspects of data science.

- Machine learning (e.g., classifiers, regressors, support vector machines, clustering techniques, data mining).
- Data visualisation, dimensionality reduction, feature extraction, network science, topological data analysis.
- Real-, complex-, quaternion-, octonion- and Clifford-valued Neural Networks, Geometric Deep Learning.
- Optimisation problems and meta-heuristic algorithms (e.g., genetic algs, particle swarm optimization algs)
- Image/signal processing and transforms, Geographical Information data.
- Analysis of ‘big data’ in mathematical data sets e.g., from algebraic geometry and mathematical physics such as Calabi-Yau data.
- Cryptography, p-adic Geometric Algebras, data compression, quantum computing.

A. What can mathematics, physics, engineering do to improve/develop learning algorithms?

B. How can learning algorithms be used to fuel research in mathematics, physics, engineering?

C. In terms of **A** or **B**, what challenges and opportunities can be envisioned from the fast ongoing advances in AI?

[1] (ye-2022) *Geometry of Deep Learning. A signal processing perspective*

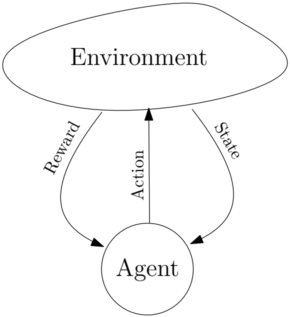
[2] (abbas-sutter-figalli-woerner-2021): *Effective dimension of machine learning models*

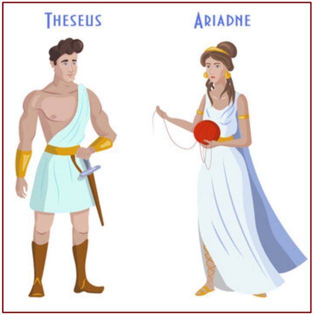
[3] (bach-2023): *Learning Theory from First Principles*

[4] (lample-charton-2019): *Deep learning for symbolic mathematics*.

[5] (davis-2019): *The Use of Deep Learning for Symbolic Integration A Review of (Lample and Charton, 2019)*

[6] (wagner-2021): "We demonstrate how by using a reinforcement learning algorithm, one can find explicit constructions and counterexamples to several open conjectures in extremal combinatorics and graph theory" (from the Abstract).





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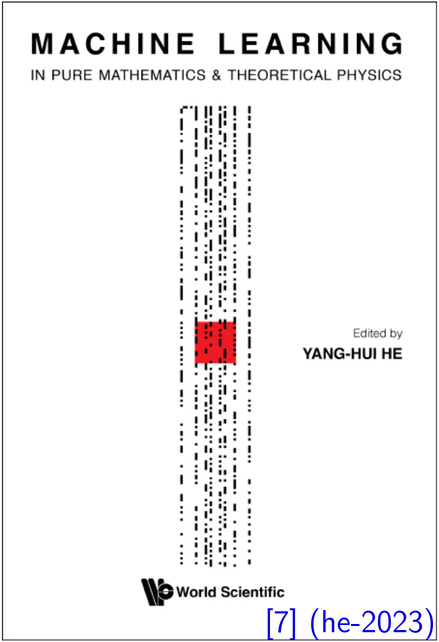
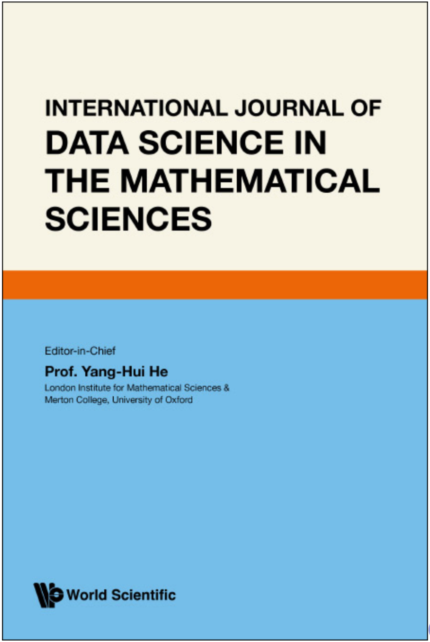

MACHINE LEARNING
IN PURE MATHEMATICS & THEORETICAL PHYSICS
Edited by
YANG-HUI HE
World Scientific

[7] (he-2023)


INTERNATIONAL JOURNAL OF
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Merton College, University of Oxford
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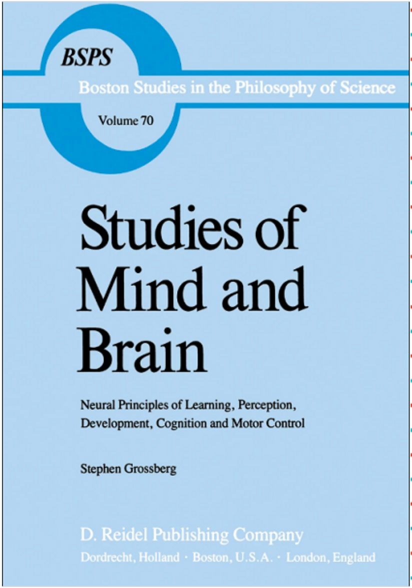

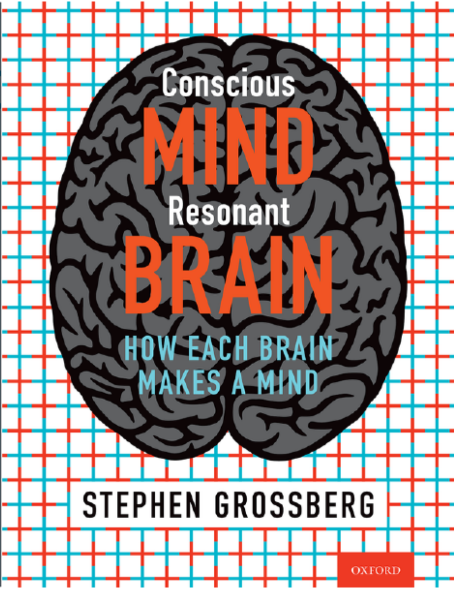

[8] (grossberg-1982) [9] (grossberg-2021)

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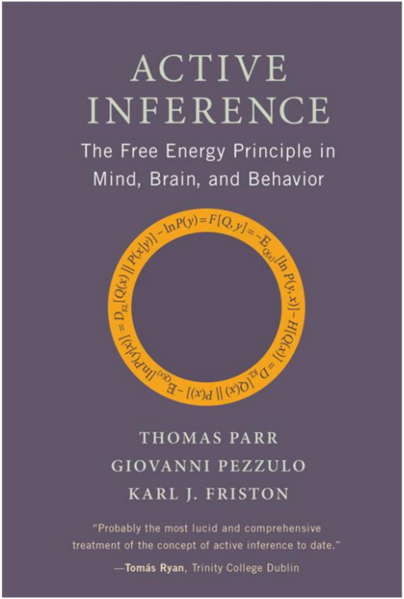

[10] (parr-pezzulo-friston-2022)

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Tutorial

A step-by-step tutorial on active inference and its application to empirical data

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ABSTRACT

The active inference framework, and in particular its recent formulation as a partially observable Markov decision process (POMDP), has gained increasing popularity in recent years as a useful approach for modeling neurocognitive processes. This framework is highly general and flexible in its ability to be customized to model any cognitive process, as well as simulate predicted neuronal responses based on its accompanying neural process theory. It also affords both simulation experiments for proof of principle and behavioral modeling for empirical studies. However, there are limited resources that explain how to build and run these models in practice, which limits their widespread use. Most introductions assume a technical background in programming, mathematics, and machine learning. In this paper we offer a step-by-step tutorial on how to build POMDPs, run simulations using standard MATLAB routines, and fit these models to empirical data. We assume a minimal background in programming and mathematics, thoroughly explain all equations, and provide exemplar scripts that can be customized for both theoretical and empirical studies. Our goal is to provide the reader with the requisite background knowledge and practical tools to apply active inference to their own research. We also provide optional technical sections and multiple appendices, which offer the interested reader additional technical details. This tutorial should provide the reader with all the tools necessary to use these models and to follow emerging advances in active inference research.

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Introduction

Active inference, and in particular its recent application to partially observable Markov decision processes (POMDPs; defined below), offers a unified mathematical framework for modeling perception, learning, and decision making (Da Costa, Parr et al., 2020; Friston, Parr, & de Vries, 2017c; Friston, Rosch, Parr, Price, & Bowman, 2018; Parr & Friston, 2018b). This framework treats each of these psychological processes, and their interactions, as interdependent forms of inference. Namely, decision-making agents are assumed to infer the probability of different external states and events in the environment – including their own actions – by combining prior beliefs with sensory input. Unlike 'passive', perceptual inference processes (e.g., inferring the presence of an external object based on patterns of light impinging on the retina), the inferences underlying decision-making are 'active', in the sense that the agent infers the actions most likely to generate preferred sensory input (e.g., inferring that eating some food will reduce a feeling of hunger). Agents also infer the actions most likely to reduce uncertainty and facilitate learning (e.g., inferring that opening the fridge will reveal available food options). This leads decision-making to favor actions that optimize a trade-off between maximizing reward and information gain. The resulting patterns of perception and behavior predicted by active inference match well with those observed empirically (e.g., see Smith et al., 2021d, 2021c, 2020b; Smith, Kuplicki, Teed, Upshaw, & Khalsa, 2020c; Smith et al., 2021e, 2020e). The neural process theory associated with active inference has also successfully reproduced empirically observed neural responses in multiple research paradigms and generated novel, testable predictions (Friston, FitzGerald, Rigoli, Schwartenbeck, & Pezzulo, 2017a; Schwartenbeck, FitzGerald, Mathys, Dolan, & Friston, 2015; Whyte & Smith, 2020). Due to these and other considerations, this framework has become increasingly influential in recent years within psychology, neuroscience, and machine learning.

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E-mail address: rsmith@laureateinstitute.org (R. Smith).
1 These authors contributed equally.

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0022-2496/© 2021 The Author(s). Published by Elsevier Inc. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

[11] (smith-friston-whyte-2022)


- Superficially, deep learning and brain modeling are pursuing a similar goal, though from quite different perspectives.
- But artificial intelligence, as in the LLMs, may be as different from biological intelligence as an Airbus 380 from an albatros.

There is an ongoing buzz on the capabilities of LLM (in particular of the Generative Pre-Trained Transformer GPT-4, 67 days old today), the likely increase of their capabilities in successive releases, and the consequences it may have in the months and years ahead: [12], [13], [14], [15]. See also Tecnological singularity.

There is every reason to expect that GPTs can be further enhanced by *cognitive implants*, that is, plugged-in to other systems that have capabilities that maybe cannot be reached by a pure GPT model.

This seems to have already happened, as you can see by visiting [16] (wolfram-2023-chatgpt) (book, paper and video), hyped as "WolframAlpha is the way to bring computational knowledge superpowers to ChatGPT".

Will this also happen with Lean? BTW, if GPT-n understands Lean, then also pure GA thanks to Eric Wieser (the winner of ICCA12 best paper, awarded during the Opening ceremony of this ICCA). N

[17] (cope-stm-2022) This paper concludes that "the submission of suspected fake research papers, also often associated with fake authorship, is growing and threatens to overwhelm the editorial processes of a significant number of journals."

[18] (brainard-2023) "Fake scientific papers are alarmingly common." "Journals are awash in a rising tide of scientific manuscripts from paper mills—secretive businesses that allow researchers to pad their publication records by paying for fake papers or undeserved authorship." "Such manuscripts threaten to corrupt the scientific literature, misleading readers and potentially distorting systematic reviews." "The recent advent of artificial intelligence tools such as ChatGPT has amplified the concern."

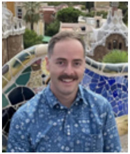
Not to speak of what such systems can and could do in the hands of other ill-intentioned people or organizations.

Session Speakers

 Carles Casacuberta is full professor of Geometry and Topology at the UB.

He has worked in algebraic topology since 1985. In 2019 he started carrying out research in the interface between topological data analysis and machine learning. He is a coauthor of several articles dealing with topology-based regularizers of neural networks and biomedical data analysis by means of persistence descriptors. He founded the Topological Machine Learning Seminar at UB, jointly with two PhD students and collaborators from the research group on Artificial Intelligence in Medicine and the Human Pose Recovery and Behavior Analysis research group from the UB's Faculty of Mathematics and Computer Science.

The main focus of this applied work is the analysis of heart diseases through magnetic resonance imaging and electrocardiogram signals. He and his group are also interested in the intrinsic dimensionality of data sets and complex networks by means of persistence summaries combined with machine learning classifiers, currently with use cases from behavioral neuroscience.

 JEREMIAH BILL is a PhD candidate in the Department of Operational Sciences at the Air Force Institute of Technology, specializing in Machine Learning and Applied Statistics.

His primary research interests involve the optimization of QNNs and their applications to various real-world datasets. Has published work in the use of heuristic optimization methods for training QNNs and a comparison of various QNN backpropagation methods. In addition to exploring the loss surfaces of QNNs, he is investigating novel applications of QNNs to real-world reinforcement learning datasets for continuous control applications.

NELSON FELIPE LOUREIRO VIEIRA is an Assistant Researcher at the Department of Mathematics of the University of Aveiro (Portugal) since 2015. Before this position, he graduated in Mathematics (Pedagogical Specialization) at the University of Aveiro in 2003, he finished his MSc and PhD in Mathematics also at the University of Aveiro in 2005 and 2009, respectively. He had a Post-Doc position at the University of Porto between 2010 and 2012. He has published several research papers, book chapter, proceedings' papers, and one book.

He has organized several scientific meetings and supervised several young students including MSc students and a Post-Doc researcher. He participates and/or has participated as a Doctoral Fellow in 1 project(s), Post-Doctoral Fellow in 1 project(s), Researcher in 6 project(s) and Responsible Investigator in 3 project(s).

His current research interests include Fractional Calculus; Hypercomplex Analysis; Fractional Clifford Analysis; Special Functions; Integral Transforms; Linear and Non-Linear Fractional ODEs; Linear and Non-Linear Fractional PDEs; Cornea Models; Clinical Image Analysis; Neural Networks; Activation Functions; CNN Architectures; Fractional Neural Networks.

 Francisco Gil Montoya received the PhD degree in evolutionary optimization techniques applied to power systems at the University of Granada, in 2009.

He is currently a member of the Engineering Department, University of Almería, where he has spent almost 20 years as a Professor, researching and teaching in areas such as power systems, energy saving, and evolutionary optimization. He has published more than 90 articles in journals, conferences, and workshops, and also has several publications in books and conference proceedings.

He is one of the creators of the geometric algebra power theory applied to power systems in the frequency and time domain.

YOLANDA VIDAL received her B.E. degree in Mathematics in 1999 and her Ph.D. degree in Applied Mathematics in 2005 from the Universitat Politècnica de Catalunya (UPC), Barcelona, Spain.

Since 2001, she has been with the Department of Mathematics and the Barcelona East School of Engineering (EEBE), at Universitat Politècnica de Catalunya, where she is currently an Associate Professor with the Control, Data, and Artificial Intelligence research group (CoDAlab).

Her research interests include structural health monitoring, condition monitoring, and fault diagnosis, with an emphasis on their specific application to wind turbines.

She is an IEEE Senior member and serves as an Editorial Board Member for international journals, such as Wind Energy (Wiley, Q1) and Engineering Applications for Artificial Intelligence (Elsevier, Q1). Has been the leader of 19 competitive projects and authored 60 journal articles, 17 book chapters, 10 books, 1 invention patent, and 112 conference articles.


EDUARDO U. MOYA is the Director of Artificial Intelligence at the General Coordination of Innovation in the State of Jalisco, Mexico (the first director of this area in the public administration in Mexico).

Some of his projects were selected by GPAI (2020)* and Global UNESCO IRCAI† (TOP-10) due to their responsible and ethical design.

He holds a PhD from CINVESTAV, a master's degree in Medical Physics from UNAM, and a Bachelor's in Physics from the University of Guadalajara. He is a member of the National System of Researchers of CONACYT level 1. In 2019, he was recognized with the Fulbright García-Robles grant to collaborate with the

Quantitative Bioimaging Laboratory at the University of Texas in Dallas and the University of Texas Southwestern Medical Center. He collaborated in deep learning research and applications at the Barcelona Supercomputing Center at the high-performance artificial intelligence group.

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* Global partnership on AI, an OCDE organization that in December 2020 issued a worldwide call focused on Areas for future action in the responsible AI ecosystem. More than 300 initiatives were submitted, 30 were selected, including #5 AI-Based Referral System (in the column of AI & SocialGood, with nine others): "A diabetic retinopathy screening program for early detection and treatment through convolutional neural networks, based on Mexican clinical guidelines, that will be implemented in three hospitals in Mexico for early detection and treatment of diabetic retinopathy."

† International Research Centre on AI under the auspices of UNESCO, Presentation of global top 100 international list of AI solutions (2022), including Artificial Intelligence-Based Referral System for Patients With Diabetic Retinopathy in Jalisco (outstanding, TOP 10/100).

 GUILLERMO GUIVANNI ALTAMIRANO ESCOBEDO

received his BS degree in Physics from the Autonomous University of Zacatecas (UAZ) in 2018. He received MS degree in Physics from the Autonomous University of San Luis Potosi (UASLP) in 2019. He is pursuing a PhD degree in Electrical Engineering from the Center for Research and Advanced Studies of the National Polytechnic Institute (CINVESTAV).

His research interests include deep learning using Clifford algebras and quantum neural networks.

Carles Casacuberta
A topological classifier to characterize brain states: When shape matters more than variance

Jeremiah Bill
The Intrinsic Dimensionality of Quaternion Neural Network Loss Landscapes

Nelson Vieira
Bessel type activation functions in Bicomplex neural networks

Francisco G. Montoya
Unraveling the geometry behind evolutionary optimization algorithms: a geometric algebra based approach
(in collaboration with Isiah Zaplana)

Yolanda Vidal
Artificial Neural Networks for Wind Turbine Predictive Maintenance

E. Ulises Moya-Sánchez
Analysis of the role of complex and quaternion Fourier transforms of images as inputs of Deep ConvNets
(in collaboration with Ulises Cortés)

Guillermo Altamirano-Escobedo
Quantum Convolutional Geometric (Clifford) Neural Network
(in collaboration with Eduardo Bayro-Corrochano)

See you on Thursday 8 in the afternoon (14:25 to 18:55), Room 2.

Thank you!!

https://web.mat.upc.edu/sebastia.xambo/99/s-icca-TML-SX.pdf

Notes

Pierre Dechant: (Curriculum redefined) Lecturer in Mathematics and Data Science at the University of Leeds.

Plenary lecture at this ICCA13 (Tue 9:00-9:45): *From Clifford spinors and ADE correspondences to characteristic multivector invariants and machine learning*.

Lecture in session 1: *Clifford algebras and ADE invariants*

Recent arXiv preprints: [19] (dechant-2022)
Root systems & Clifford algebras: from symmetries of viruses to E_8 & ADE correspondences,
[20] (dechant-he-heyes-hirst-2022)
Cluster Algebras: Network Science and Machine Learning,
[21] (cheung-dechant-he-heyes-hirst-li-2022)
Clustering Cluster Algebras with Clusters.

YANG-HUI HE: Hé Yáng Huij. Mathematical Physicist, at the London Institute of Mathematical Sciences, Lecturer at Merton College, Oxford. Prolific scientist. In his youtube videos, his demeanor has touches of a foreteller, of an ageless prophet. I'll say a bit more about He in a moment.

ISIAH ZAPLANA: Maria Zambrano postdoctoral fellow on robotics and mathematical optimization at the Institute of Industrial and Control Engineering of the UPC (BarcelonaTech).

Recent arXiv preprint: [22] (zaplana-hadfield-jlasenby-2022) *Singularities of serial robots: Identification and distance computation using geometric algebra*.

I'll return to his work below.

P

Cover of the ICCA3 Proceedings (Deinde, Belgium, 1993). It collects 45 papers, of which 22 are on Clifford Analysis. It is a representative image of the early ICCA's. Afterwards the Proceedings morphed into Topical Collections, not only for ICCA, but also for other conferences.

2008 (8th IMECC-UNICAMP, Campinas, Brazil): [23] (icca2008) (10 papers)

2017 (11th Ghent University, Belgium): [24] (icca2017) (6 papers).

P

"In this paper [[4] (lample-charton-2019)], we show that [supervised learning algorithms] can be surprisingly good at more elaborated tasks in mathematics, such as symbolic integration and solving differential equations. We propose a syntax for representing mathematical problems, and methods for generating large datasets that can be used to train sequence-to-sequence models. We achieve results that outperform commercial Computer Algebra Systems such as Matlab or Mathematica" (from [4], Abstract).

[6] (wagner-2021): "An interesting feature is that in some cases the learning algorithm does not produce directly a counterexample but graphs which are close to refuting the conjecture; these graphs have a special structure and give a very clear indication about where to search for counterexamples" (review of [5] in NL01, page 20).

P

The juxtaposition of ML and "pure mathematics and theoretical physics" may first appear as contradictory in terms. The rigours of proofs and derivations in the latter seem to reside in a different world from the randomness of data and statistics in the former. Yet, *an often under-appreciated component of mathematical discovery, typically not presented in a final draft, is experimentation*: both with ideas and with mathematical data. Think of the teenage Gauss, who conjectured the Prime Number Theorem by plotting the prime-counting function, many decades before complex analysis was formalized to offer a proof.

Can modern technology in part mimic Gauss's intuition? The past five years saw an explosion of activity in using AI to assist the human mind in uncovering new mathematics: finding patterns, accelerating computations, and *raising conjectures via the machine learning of pure, noiseless data*. The aim of this book, *a first of its kind*, is to collect research and survey articles from experts in this emerging dialogue between theoretical mathematics and machine learning.

It does not dwell on the well-known multitude of mathematical techniques in deep learning, but focuses on the reverse relationship: how machine learning helps with mathematics. Taking a panoramic approach, the topics range from combinatorics to number theory, and from geometry to quantum field theory and string theory. Aimed at PhD students as well as seasoned researchers, each self-contained chapter offers a glimpse of an exciting future of this symbiosis.

From the Preface, final paragraph: “I sincerely hope that this volume offers the first glimpse onto a fertile land, a cross-disciplinary world of mathematicians, physicists and computer scientists. This nascent collaboration between machine learning and pure mathematics as well as theoretical physics, a taste of the spirit of which we hope to capture here, will undoubtedly continue to flourish.”

Note: Thanks to WorldScientific for providing information of this “first of its kind” book, including the contents, the preface of the author, and the short profiles of the authors. P

I learnt from the book on the left, [8] (grossberg-1982), through the HESTENES' paper [25] (hestenes-1987-brain) (*How the brain works: the next great scientific revolution*), of which I would like to quote the following impressive judgment:

"In spite of the enormous complexity of the human brain, there are good reasons to believe that only a few basic principles will be needed to understand how it processes sensory input and controls motor output. In fact, the most important principles may be known already! These principles provide the basis for a definite mathematical theory of learning, memory, and behavior."

This led me to search for GROSSBERG's works and discovered the book on the right, [9] (grossberg-2021), which in my view comes close to a *Philosophiæ Neuronalis Principia Mathematica*. Corresponding with DAVID HESTENES on this matter on May 21st, he sent me the following comment on [9] (grossberg-2021) (a dustcover 'blurb', he said):

"This book deserves its billing as the PRINCIPIA of MIND.

In a lifetime of dedicated research Grossberg has single-handedly elevated the psychophysics and psychology pioneered by Herman von Helmholtz and William James into a comprehensive mathematical theory of brain and behavior with profound implications and strong empirical support.

This book provides a nontechnical introduction to basic brain mechanisms involved in all aspects of human perception and behavior with a broad survey of implications across the intellectual spectrum. Thus, it can serve as a guide to serious study by students and experts alike.

Indeed, it can be recommended as a textbook to penetrate the entire college curriculum with reliable common knowledge about the human condition and its possibilities."

Born in 1939, Stephen Grossberg is Wang Professor of Cognitive and Neural Systems at Boston University, Professor Emeritus of Mathematics & Statistics, of Psychological & Brain Sciences, and of Biomedical Engineering. Google scholar: 1034 items; 80464 citations (8439 since 2018); h-index 131 (40 since 2018); i10-index 389 (182 since 2018).

P

Among the many people that worry about the future are the signatories of Satement on AI Risk (count about 2 days ago: 42 from DeepMind, 21 from OpenAI, 11 from Anthropic, 7 from UC Berkeley, 4 from Chalmers University of Technology, 2 from Google-Alphabet, 0 from Meta or Galactica, ...). The statement is:

"Mitigating the risk of extinction from AI should be a global priority alongside other societal-scale risks such as pandemics and nuclear war."

Two prominent instances: GEOFFREY HINTON, in [26] (hinton-2023-5-11):

"People are just blind to this danger", "It wasn't really foreseeable this stage, and until very recently I thought this existential crisis was a long way off, so I don't really have any regrets about what I did":

And Max Tegmark, in interview [27]: "...what we have now is still short or what is called AGI, this Holy Grail of making machines that can outsmart us on basically all job tasks, but we are getting there at express speed. It turned out that it was much easier to build this than people thought 10 years ago. Ten years ago people thought ah 50 years, maybe 30 years, maybe way longer ... now you should see a lot of the top experts in the field giving much much shorter timelines. We already have arguable passed the mastery of the human language for example, and so I'm hoping this will make a lot of policy makers realize that this is not science fiction. Intelligence is not something mysterious that can only exist in human brains; it's something that we can also build and when we can build it wecan also very easily build things which are satraightly beyond us as far as we are beyond insects, right, and obviously we're building this we should build build AI for humanity by humanity and not for the purpose of machines having a great time later on, right, so making sure we really give ourselves a time, and to make sure we control these machines, or make sure at least that they have our values and do what we want. That is something more important than any other choice that humanity is making right now."

There are also prominent critical voices of this 'doom mood', which instead emphasize the positive aspects. For instance KyungHyun Cho (his PhD thesis [28] (cho-2014) and the paper [29] (bahdanau-cho-bengio-2014) have been key advances for the LLM technology): see [30] (cho-2023).

The question from outside is qui prodest? (who benefits?).

FirstPost, June 1, 2023: "Danish prime minister Mette Frederiksen was giving a traditional speech as Parliament gets ready to close for the Summer. She takes a pause in the middle to announce that the speech was not written by her or any other human. But the artificial intelligence tool ChatGPT. P

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