Geometric bio-inspired Deep Learning

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16/07/2019

- **Why to go beyond CNN**
- **A new kind of layer**
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Main reference: *A bio-inspired monogenic CNN layer for illumination-contrast invariance* (submitted June 2019). E. Ulises Moya-Sánchez, Sebastián Salazar Colores, Sebastià Xambó-Descamps, Abraham Sánchez and Ulises Cortés.

Why to go beyond CNN

- no suitable theory of CNNs design is available.
- the learning of an *invariance response* may fail even with very deep CNNs or by large data augmentations in the training.
- DL cannot guarantee capabilities of the mammalian visual systems such as lighting changes and rotation equivariance.

- *Learning algebraic varieties from samples* (Real algebraic varieties and computer algebra) breiding-sturmfelds-et2-2018 [17]
- *Group equivariant convolutional networks* cohen-welling-2016 [33]
- *Geometric deep learning* bronstein-bruna-lecun-szlam-vandergheynst-2017 [18]
- *Understanding deep convolutional nets* mallat-2016 [78]
- *Convolutional networks for spherical signals* cohen-geiger-kohler-welling-2017 [32]
- *Low-level image processing with the structure multivector* (felsberg-2002 [43])

Our strategy for exploring "Geometric Deep Learning" is to use **Geometric Calculus** (GC) in the sense of:

- *Clifford Algebra to Geometric Calculus* hestenes-sobczyk-1984 [54] (**bible of GC**)

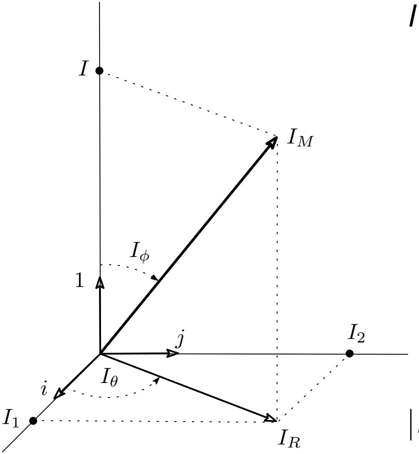
- *Applications to relativity, robotics and molecular geometry* lavor-xambo-zaplana-2018 [70]

- The main strong points for this advance are the long history of achievements in a great variety of fields (see [120], §6.4)

- Additional bonus: The representation of the signals occurs in a higher dimensional space and hence they provide naturally a stronger discrimination capacity.

- There is a well-developed theory of GC wavelets with the potential to be applied to DL much as scalar wavelets are used in current DL techniques: [82], [29], [59].

A new kind of layer



$$I = I_{\text{mono}} * \text{LogGabor}$$

$$I_M = I + I_{M'}, \quad I_{M'} = iI_1 + jI_2$$

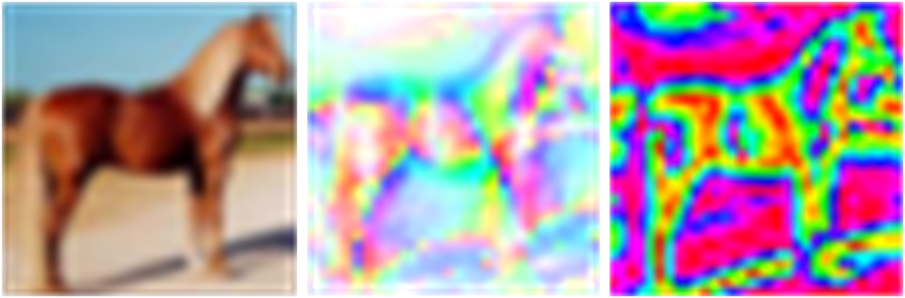
$$I_1 = I * h_1, \quad I_2 = I * h_2$$
$$h_1(x,y) = -\frac{x}{2\pi\left(x^2+y^2\right)^{3/2}}$$
$$h_2(x,y) = -\frac{y}{2\pi\left(x^2+y^2\right)^{3/2}}$$

$$|I_M|^2 = |I|^2 + |I_{M'}|^2 = |I|^2 + |I_1|^2 + |I_2|^2$$

$$I_\phi = \operatorname{atan2}\left(\frac{I}{|I_R|}\right) \; I_\theta = \operatorname{atan}\left(\frac{-I_2}{I_1}\right)$$

$$HSV_\phi = (H,S,V) = (I_\phi, |I_R|, 1), \quad HSV_\theta = (H,S,V) |= (I_\theta, |I_R|, 1)$$

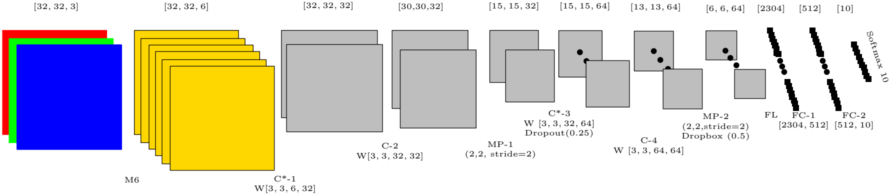
$$\text{RGB}_\theta,\ \text{RGB}_\phi$$

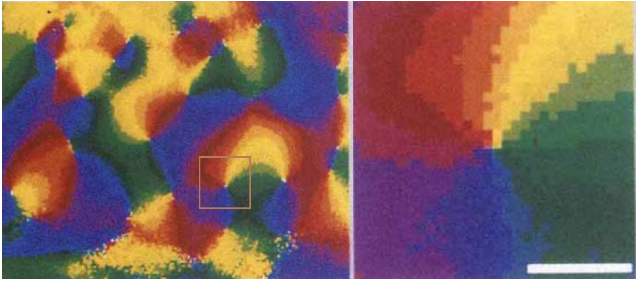


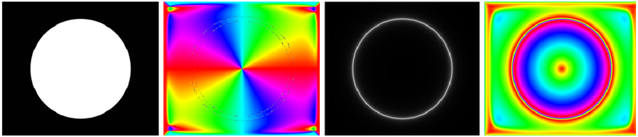
(a) (b) (c)

(a) RGB input image. (b) RGB_θ. (c) RGB_ϕ. M6 $=$ (b) $+$ (c).

Source computations:
https://github.com/asp1420/monogenic-cnnillumination-contrast.


[32, 32, 3]
[32, 32, 6]
[32, 32, 32]
[30,30,32]
[15, 15, 32]
[15, 15, 64]
[13, 13, 64]
[6, 6, 64]
[2304]
[512]
[10]
Softmax 10
M6
C*-1
W[3, 3, 6, 32]
C-2
W[3, 3, 32, 32]
MP-1
(2,2, stride=2)
C*-3
W [3, 3, 32, 64]
Dropout(0.25)
C-4
W [3, 3, 64, 64]
MP-2
(2,2,stride=2)
Dropbox (0.5)
FL
FC-1
[2304, 512]
FC-2
[512, 10]

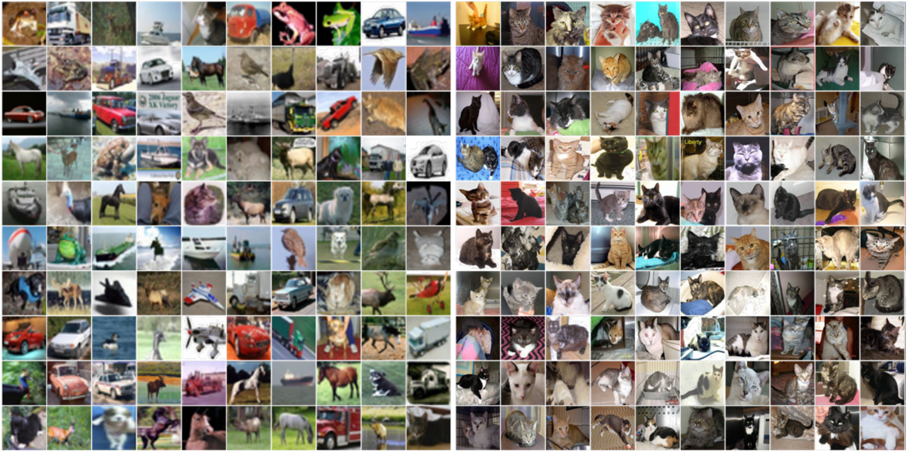




Data and experimental setup

	CIFAR-10	Dogs and Cats
Training set	36 000	2 400
Validation set	12 000	800
Test set	12 000	800
Total of images	60 000	4 000
Input shape	[32, 32, 3]	[150, 150, 3]

(a) 100 RGB images from the CIFAR-10 dataset
(b) 100 images from the Dogs and Cats dataset.



(a) (b)

(a) 100 RGB images from the CIFAR-10 dataset
(b) 100 images from the Dogs and Cats dataset.


d_0


d_1

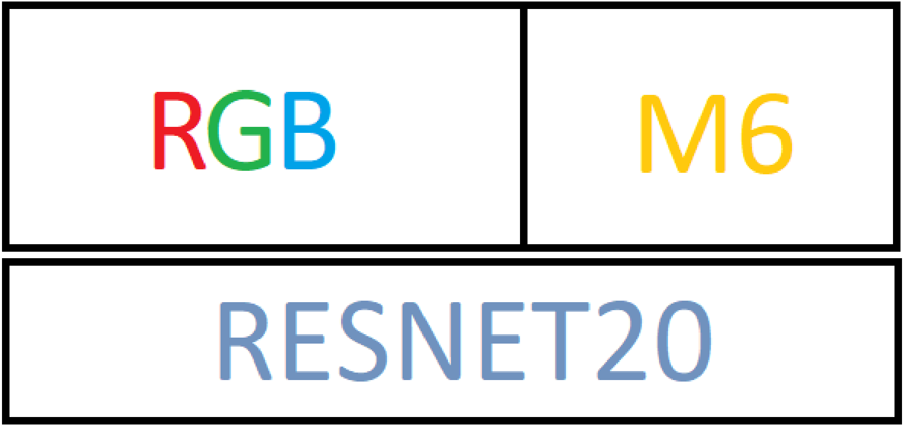

d_2


d_3

CNN	
Trained	Tested
d_0	d_0, d_1, d_2, d_3
d_1	d_0, d_1, d_2, d_3
d_2	d_0, d_1, d_2, d_3
d_3	d_0, d_1, d_2, d_3

CIFAR-10	
I	$[32, 32, n_3 = 3]$
$M6$	$[32, 32, n_3 = 6]$

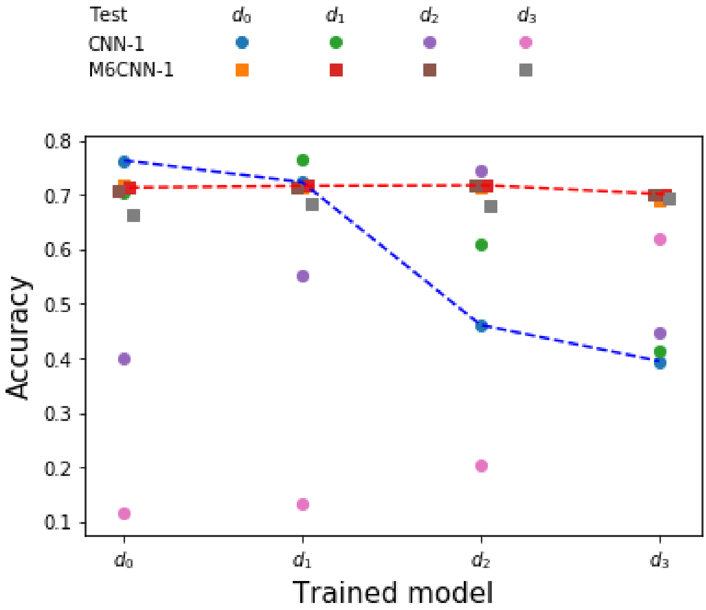
$\mathbf{x} \to \mathbf{x}'$	W	$\mathbf{x}'$
C*-1	$[3, 3, n_3, 32]$	$[32, 32, 32]$
C-2	$[3, 3, 32, 32]$	$[30, 30, 32]$
MP-1	$[2, 2, s = 2]$	$[15, 15, 32]$
C*-3	$[3, 3, 32, 64]$	$[15, 15, 64]$, Dropout (0.25)
C-4	$[3, 3, 32, 64]$	$[13, 13, 64]$
MP-2	$[2, 2, s = 2]$	$[6, 6, 64]$ Dropout (0.25)
FL		$[2304]$
FC-1	$[2304, 512]$	$[512]$
FC-2	$[512, 10]$	$[10]$ Dropout (0.5)
SMAX		$[10]$

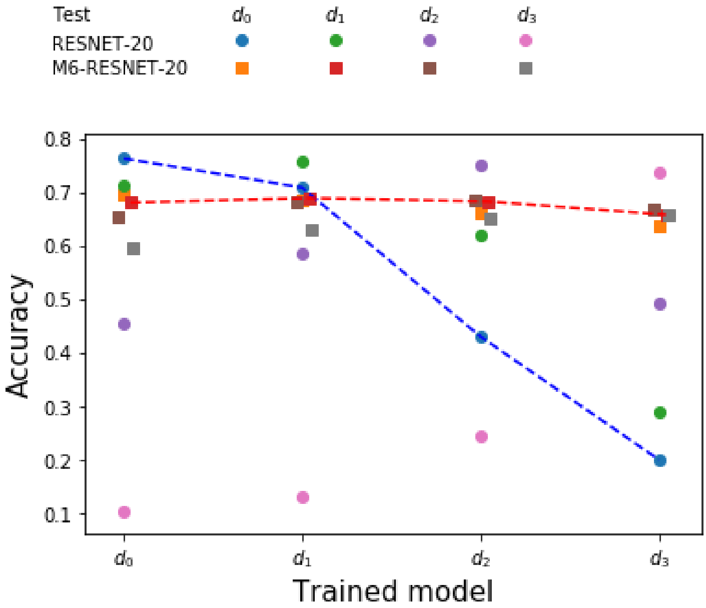


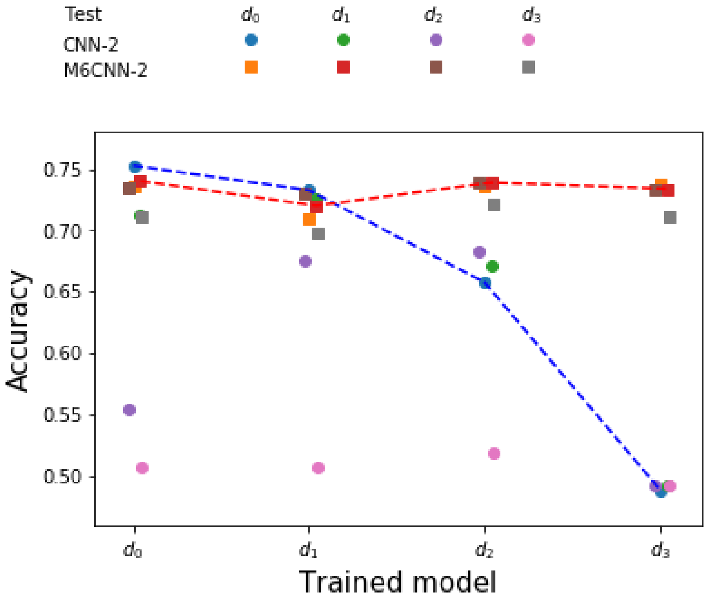
K. He, X. Zhang, S. Ren, and J. Sun, *Identity mappings in deep residual networks*, in European Conference on Computer Vision, pp. 630-645.

Cats and Dogs		
I	$[224, 224, n_3 = 3]$	
$M6$	$[224, 224, n_3 = 6]$	
$\mathbf{x} \to \mathbf{x}'$	W	$\mathbf{x}'$
C*-1	$[3, 3, n_3, 32]$	$[224, 224, 32]$
MP-1	$[2, 2, s = 2]$	$[112, 112, 32]$
C-2	$[3, 3, 32, 32]$	$[110, 110, 32]$
MP-2	$[2, 2, s = 2]$	$[55, 55, 32]$
C-2	$[3, 3, 32, 64]$	$[53, 53, 64]$
MP-2	$[2, 2, s = 2]$	$[26, 26, 64]$
FL		$[43264]$
FC-1	$[438264, 64]$	$[64]$ Dropout (0.5)
SMAX		$[2]$

Main results


Test
d_0
d_1
d_2
d_3
CNN-1
M6CNN-1
Accuracy
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
d_0
d_1
d_2
d_3
Trained model

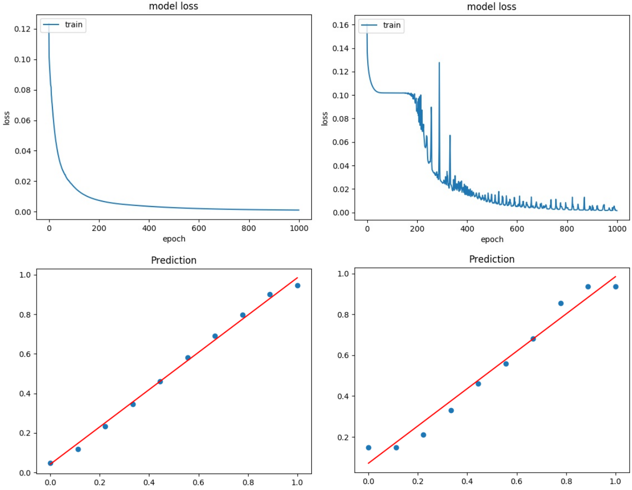

Test
d_0
d_1
d_2
d_3
RESNET-20
M6-RESNET-20
Accuracy
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Trained model


Test
d_0
d_1
d_2
d_3
CNN-2
M6CNN-2
Accuracy
0.75
0.70
0.65
0.60
0.55
0.50
Trained model

Conclusions and outlook

- Geometric calculus has the potential to articulate *novel and promising researches* in deep learning (GCDL).
- We have designed a bio-inspired front layer for CNNs that processes a monogenic signal by extracting phase and orientatation signals and assembling them in a HSV space.
- The experimental results with two different datasets and three CNNs confirm that the accuracy gained by using our layer has a substantially more robust performance when faced with severe illumination changes than the same nets without such a front layer.

To continue the trail walked in this research by developing a front layer for CNNS that is resilient when faced with other transformations of the images, as for instance rotations or even small deformations.



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