Stein Arild Strømme (1951–2014) *in memoriam*

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Figure 1. Stein A. Strømme as Director of the Mathematics Department, University of Bergen

Professor Stein Arild Strømme*, of the* University of Bergen *and associated with the* Oslo Centre of Mathematics and Applications*, passed away on 31 January this year at the age of 62. A well known specialist in algebraic geometry, in particular in the areas of moduli spaces, intersection theory and enumerative geometry, he was also Editor of* Acta Mathematica (1997–2000) *and a committed leader on the international stage with endeavours such as the* Europroj *project, the* Sophus Lie Center *at Nordfjordeid and the* Mittag-Leffler Institute *at Djursholm (Stockholm).*

The Maple package SCHUBERT*, developed jointly with* Sheldon Katz*, accredited him as a computational wizard. The aim of this article is to present a more detailed overview of these accomplishments, the milieu in which they took form and the visible influence on current undertakings.*

A masterpiece (and a big moment for mirror symmetry)

This high praise for *The number of twisted cubic curves on the general quintic threefold* [21] by one reviewer (Bruce Hunt) is aptly asserted in the opening sentence of his excellent review.[1] A reviewer (Susan Colley) of [23], which is the final version of [21], justly refers to it as "a *tour de force* of enumerative geometry".[2] She also notes that in *Bott's formula and enumerative geometry* [24] (which is clearly another *chef d'œuvre*) "the authors have obtained the number 317 206 375 of twisted cubics on a general quintic hypersurface in $\mathbf{P}^4$ in another way, namely by using a residue formula of Bott. This technique avoids some of the intricate and involved intersection-theoretic calculations used in the paper under review." The significance of these works, which goes far beyond their mathematical and computational virtuosity, is rooted in the context in which they were produced. In 1990, a principle of 'mirror symmetry' for Calabi-Yau manifolds was proposed by string theorists Brian Greene and Ronen Plesser.[3] This principle, together with other insights of the late 1980s relating Calabi-Yau manifolds and conformal field theories, was used by Philip Candelas and his collaborators (Xenia de la Ossa, Paul Green and Linda Parkes) to predict the number N_d of rational curves of any degree d contained in a generic quintic 3-fold in $\mathbf{P}^4$.[4] What Strømme and Ellingsrud did was an independent computation of N_3 according to the (algebraic geometry based) standards of enumerative geometry at that time.[5] At first (May 1991), their result disagreed with the physicists' prediction but after correcting a bug in their program they got the same answer and graciously recognised the error by sending the famous "Physics wins!" message (June 1991). If it was true that the numbers N_d, for $d \geq 4$, seemed beyond the reach of current enumerative geometry methods, the retrospect perspective on the positive significance of the computation of N_3 has been emphasised by leading researchers, as for example by Shing-Tung Yau in [68] (p. 170, my emphasis):

> This proved to be *a big moment for mirror symmetry*. The announcement of Ellingsrud and Strømme not only advanced the science of mirror symmetry, but also helped change attitudes toward the subject [... mathematicians] now came to realise there was something to be learned from the physicists after all.

In [31], Brian Greene had already presented these new connections between mathematics and physics with the words (p. 262, my emphasis):

> For quite some time, physicists have 'mined' mathematical archives in search of tools for constructing and analyzing models of the physical world.

1 MR1191425 (94d:14050)

2 MR1345086 (96g:14045)

3 B. R. Greene and M. R. Plesser. *Duality in Calabi-Yau Moduli Space*. Nuclear Physics B 338 (1990), 15-37. In the case of a Calabi-Yau 3-fold, Poincaré duality and Hodge duality imply that the Hodge numbers $h^{1,1}(X)$ and $h^{1,2}(X)$ determine all the other $h^{p,q}(X)$. The 'mirror symmetry' conjecture, made on the grounds of a 'physical duality', was that there ought to exist a Calabi-Yau 3-fold $\bar{X}$ such that $h^{1,1}(\bar{X}) = h^{1,2}(X)$ and $h^{1,2}(\bar{X}) = h^{1,1}(X)$.

4 P. Candelas, X. de la Ossa, P. Green, L. Parkes. *A pair of Calabi-Yau manifolds as an exactly soluble superconformal field theory*. Nuclear Physics B 359 (1991), 21–74.

5 $N_1 = 2875$ was determined by H. H. Schubert, [52]; $N_2 = 609250$, by S. Katz, [36].

> Now, through the discovery of string theory, physics is beginning to repay the debt and *to provide mathematicians with powerful new approaches to their unsolved problems*. String theory not only provides a unifying framework for physics, but it may well forge an *equally deep union with mathematics as well*.

Among the more striking cases of 'mining', let me just mention the 'eightfold way' approach (Murray Gell-Mann and Yuval Ne'eman) to the problem of classifying elementary particles. This happened about 30 years before the discovery of mirror symmetry and it involved a few of the representations of SU(3) that were described by Hermann Weyl over 30 years before that.[6] The eightfold way schemes somehow belong to the prehistory of string theory, for they were the first step on the way to building the so-called standard model (early 1970s), which in turn spawned a new fertile era in the relations between mathematics and physics, with leading figures such as Sir Michael Atiyah, Simon Donaldson and Edward Witten. We still have a bit more to say about this in a later section.

Small world[7]

A quick glance at the Proceedings of the Enumerative Geometry Conference Sitges 1987 [65] reveals that while there was one Scandinavian mathematician for every eight participants, they actually contributed nearly one third of the lectures and one third of the published papers. This is rather remarkable when contrasted with the participation in the previous, more general, conference Sitges'83 Week of Algebraic Geometry [4] in which there were just four Scandinavians[8] among a total of 71 participants, only two of them (Ulf Persson and Ragni Piene) as speakers.

The explanation of these figures is actually quite simple. The 1983 conference was on algebraic geometry in a general sense. It was the first conference organised in Barcelona on that topic and the invited speakers belonged to a number of specialties. The only enumerative geometer, for instance, was Ragni Piene, a student of Steven L. Kleiman (she got her PhD in 1976). By the way, the title of her contribution was *On the problem of enumerating twisted cubics* ([4], pages 329–337), a theme that is closely related to problems that were to interest Stein A. Strømme (as we will see later). But in the four years up to 1987, a small and enthusiastic enumerative geometry group was formed in Barcelona and it was decided that the second algebraic geometry conference would be focused on that area (including intersection theory). The high participation of Scandinavians on that second occasion simply reflected the fact that in those countries there were more people interested, or expert, in the conference topic than in any other country.

In the Sitges'87 conference, Strømme showed some of his cards: he lectured on the Chow ring of a geometric quotient, the substantive fruit of another collaboration with Ellingsrud which was later published in the Annals of Mathematics [20].

But it was actually two years earlier that I met Strømme for the first time. The setting was the conference on space curves held in Rocca di Papa, organised by Franco Ghione, Christian Peskine and Edoardo Sernesi (Springer LNiM 1266, 1987), and it happened through the good offices of Kleiman. I had proposed to him to go over some ideas about checking Schubert's 'big number' (number of twisted cubics that are tangent to 12 quadrics in $\mathbf{P}^3$) and he immediately arranged a few discussions of the three of us, which led to the paper [40].

The scientific participation of Strømme, however, was far larger than his part in [40], for he published two more papers in the same proceedings: [55] and [15]. The latter, in collaboration with Ellingsrud and Ragni Piene, develops a nice geometric technique for compactifying the space of twisted cubics, an insight that six years later was a keystone for the first computation of the number N_3 explained in the previous section.

Let me also say that Kleiman, who might be counted as Scandinavian,[9] submitted a wonderful paper on multiple-point formulas to Sitges'87: *Multiple-point formulas II: the Hilbert scheme* (38 pages). This was the long awaited sequel to the paper *Multiple-point formulas I: Iteration* (37 pages) that was published in Acta Mathematica in 1981.

The SCHUBERT package

The SCHUBERT Maple package [39], written by Stein A. Strømme and Sheldon Katz, was released in 1992 and was maintained, according to its website,[10] until 2011, with revisions and updates "to reflect new versions of Maple" by Jan-Magnus Økland. The package supports computations in intersection theory and enumerative geometry. It was frozen at release 0.999 but it may still be instructive to reflect on how its authors could achieve so much power with a remarkably compact code.[11] The somewhat technical digression that follows may be helpful for those interested in ideas about how such a system is built or extended, or even ported to other platforms.

To highlight a few characteristic features of the package in a suitably generic setting, I will glean a few ideas from [66], which will be referred to as SCHUBERT-WIT. It is not required to know anything about the interface WIT, for here we only rely on its rather self-explanatory pseudo-code-like expressiveness. This may also help those interested in getting acquainted with the Sage package CHOW (about which you will find a bit more at the end of this section).

One of the main ideas in SCHUBERT-WIT is that its objects are implemented as instances of `Class` definitions (or types). A `Class` definition can be thought of as a list of pairs `k : T`, where `k` denotes an identifier (a key) and `T` a type. It follows that an object can be construed as a list of pairs `k : t`, where

6 H. Weyl, *Gruppentheorie und Quantemechanik* (1928). See also the book *Essays in the history of Lie groups and algebraic groups*, by Armand Borel, especially Chapter III (History of Mathematics, Vol. 21, AMS, 2001).

7 Stolen from the title of David Lodge's 1984 novel (published by Penguin Books in 1995).

8 Audun Holme, Trygve Johnsen, Ulf Persson and Ragni Piene.

9 On account of being married to Beverly, who is Danish, but also because of the many shared undertakings with mathematicians from those countries.

10 http://stromme.uib.no/schubert/.

11 The length of the package file is 142 KB. It has about four thousand lines, of which more than half are devoted to help texts and examples.

t is an object of type T and where k runs through some subset of the Class definition keys. Objects are dynamic, in the sense that the number of keyed entries can be enlarged during a computation. The types T can be types of the underlying language (like Identifier, Integer, Polynomial, List or Vector) or other SCHUBERT-WIT types, such as Sheaf, Variety or Morphism.

Let us look at some simple examples (we refer to [56] or [64] for a quick introduction to basic concepts about intersection theory and Chern classes). The type Sheaf, for instance, is defined with the following code:

```
let Sheaf = Class
    rk : Element(Ring)
    ch : Vector
```

This means that the representation of a sheaf (an instance of Sheaf) is like a table with two entries, one with key rk (for the rank, possibly in symbolic form) and another with key ch (for the Chern character).

In order to work with sheaves, we first need some constructors, that is, functions that deliver a sheaf out of some suitable data. The simplest situation is when we know the Chern character x = [x.1,...,x.n] or, more generally, that the Chern character has the form pad(x,d), where d is an integer (a cut-off dimension) and pad(x,d) is [x.1,...,x.d] if d ≤ n or [x.1,...,x.n, 0,...,0] otherwise, with $d - n$ zeros. If these conditions can be assumed then we can define the constructor SH as follows (note the three different call forms):

```
SH(r:Element(Ring), x:Vector, d:Nat) :=
   instance Sheaf
       rk = r
       ch = if x==[] then [0] else pad(x,d)
SH(r:Element(Ring), x:Vector) :=
    SH(r,x,length(x))
SH(r:Element(Ring), x:Vector, X:Variety) :=
    SH(r,x,dim\X)
```

The expression dim\X yields the dimension of X. Indeed, as we will see below, one of the keys of the type Variety is dim, and the dimension of a variety X can be extracted either as X(dim) or dim\X (the operator \ swaps its operands and applies the second to the first).

In general, however, the data that we will have at our disposal for the construction of a sheaf is a (symbolic) vector of Chern classes c = [c.1,...,c.n] (Chern vector) and a cut-off dimension d for the Chern character. In that case we can use the following constructors:

```
sheaf(r:Element(Ring), c:Vector, d:Nat) :=
   SH(r,c2p(c,d))
sheaf(r:Element(Ring), c:Vector, X:Variety) :=
   SH(r,c2p(c,dim\X))
sheaf(r:Element(Ring),c:Vector) :=
   SH(r,c2p(c))
```

Here c2p(c,d) takes care of converting, using basic formulas from intersection theory, the vector of Chern classes to the Chern character vector, padded to the cut-off dimension d.

To construct vector bundles (locally free sheaves), we can use calls to sheaf that take advantage of additional properties of their Chern classes:

```
bundle(n:Element(Ring), c:Name, d:Nat) :=
   if is?(n,Integer) & n>=0
   then sheaf(n,vector(c,min(n,d)),d)
   else sheaf(n,vector(c,d))
bundle(n:Element(Ring), c:Name, X:Variety) :=
   bundle(n,c,dim\X)
bundle(n:Element(Ring), c:Name) :=
   bundle(n,c,DIM)
```

It is important to note that an expression such as vector(u,3), where u is a symbol, yields the symbolic vector [u1,u2,u3]. In the case that the rank data is a non-negative integer, the span of the Chern vector is at most min(n,d) and we use this fact in the first call. In the third call, DIM refers to the (current) default dimension.

In a similar way, varieties are objects of type VAR = Variety = Manifold, which is defined as follows:

```
let VAR = Class
    dim : Integer
    kind : Identifier
    gcs : Vector # of generating (Chow) classes
    degs : Vector # codimensions of the gcs
    monomials : List
    monomial_values : Table
    pt : Polynomial
    relations : Vector
    basis : List
    dual_basis : List
    tan_bundle : Sheaf
    todd_class : Vector
```

Among the several constructors of varieties, a simple illustration is given by the case of projective spaces (note the dynamic construction of the final object):

```
projective_space(n:Integer, h:Name) :=
   P=instance VAR
     dim =n
   P(kind) = if n==1 then _projective_line_
             elif n==2 then _projective_plane_
             else _projective_space_ end
   P(gcs)=[h]
   P(degs)=[1]
   P(relations)=[h^(n+1)]
   P(monomials)={{h^j} with j in 1..n}
   P(monomial_values)={h^n : 1}
   P(pt)=h^n
   P(tan_bundle)=sheaf(n,
     [binomial(n+1,j)*h^j with j in 1..n],P)
   P(todd_class)=todd_vector(P(tan_bundle),P)
   return P
```

The symbol h stands for the rational class of a hyperplane of the projective space of dimension n. Hence gcs is the vector [h] (a single component) and degs is [1]. There is a single relation ($h^{n+1} = 0$), and $1 = h^0, h^1, \ldots, h^n$ are all the nonzero monomials. The judgments pt = h^n and h^n : 1 express the fact that the intersection of n general hyperplanes is a point and that a point has degree 1.

A type such as VAR can be extended in order to define more specialised types. A nice example is the type GRASS = Grassmannian:

```
let GRASS = Class from VAR
    tautological_bundle : Sheaf
    tautological_quotient : Sheaf
```

This means that a grassmannian is a variety with two additional keys, one for the tautological bundle and another for the tautological quotient. As to constructors of grassmannians, they can be defined in a rather straightforward manner by using suitable formulas for the values of the different keys.

As an example of all this, here is a lovely script that computes the number of lines (27) on a generic cubic surface in 3-dimensional projective space:

```
G=grassmannian(1,3,c);
S=tautological_bundle\G;
F=symm(3,dual(S));
c4=chern(rk\F,F)
integral(G,c4)
```

Finally, let us consider the type MOR = Morphism:

```
let MOR = Class
    source : VAR
    target : VAR
    dim : Integer
    upperstardata : Table
    kind : Identifier
```

The key upperstardata is assigned a Table. For a given morphism f (an instance of MOR) the entries of the table upperstardata\f have the form x : u, where x runs trough gcs\target\f and u is the expression of f*x in terms of gcs\source\f.

The types MOR and VAR can be used to define the type BLOWUP:

```
let BLOWUP = Class from VAR
   exceptional_class : Element(Ring)
   blowup_map : MOR
   blowup_locus : VAR
   locus_inclusion : MOR
```

This class includes the ingredients that are involved when we blow up a variety X (this is captured in the first line) along a subvariety Y (blowup_locus). The locus_inclusion stands for the inclusion $Y \xrightarrow{i} X$ and blowup_map for the projection map $\widetilde{X} \xrightarrow{f} X$. The exceptional_class stands for the rational class of the exceptional divisor. Sure enough, we have a constructor blowup(i:MOR, e:Name, f:Name) that binds i to the locus inclusion, f to the blowup map and e to the exceptional class.

As an illustration, let us look at a script that computes the number (3264) of conics in a plane that are tangent to five smooth conics in general position:

```
P5=projective_space(5,H);
S =projective_space(2,h);
i=morphism(S,P5,[2*h]);
X=blowup(i,e,f);
integral(X,(6*H-2*e)^5)
```

Here P5 is the projective space of conics in a plane (for background, see, for example, [5]). The set of conics tangent to a given conic is a sextic hypersurface and hence its rational class is $6H$. These hypersurfaces all contain the surface S of double lines but their proper transforms on the blowing up of $\mathbf{P}^5$ along S do not intersect on the exceptional locus. As it turns out that the class of any of these proper transforms is $6H - 2e$, the number we are seeking is the degree of the intersection $(6H - 2e)^6$.

After the lecture [66], I learned that D. Grayson, M. Stillman, D. Eisenbud and S. A. Strømme were working on SCHUBERT2, a port of SCHUBERT to Macaulay2. But this project seems to have been displaced later on by CHOW, a Sage package written in Singular by M. Lehn and C. Sorger.[12] It seems

12 See http://www.math.sciences.univ-nantes.fr/~sorger/chow/html/ for details. The user manual can be downloaded from this site.

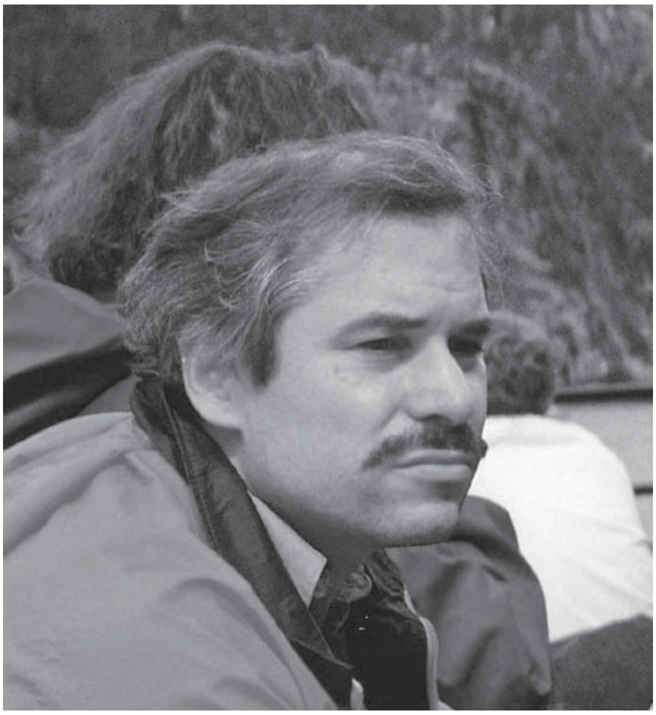

Figure 2. Sheldon Katz, coauthor of the SCHUBERT Maple package. Dyrkolbotn Conference on Mirror Symmetry, 1993

clear that Strømme's illness diminished the time and energy he could devote to the project. One of the entries in his diary reads:

> Sunday, 24 Nov 2009: Getting stronger with each passing day. This morning I was able to go to church for the first time after surgery. I could even play the violin with the worship team like before :-) Looking forward to getting started with radiation therapy on Tuesday.

And Sheldon Katz, in a message he sent me on 2 February 2014, wrote:

> Stein Arild was very courageous and inspiring to many people while he was fighting his illness these past few years. He was diagnosed in the middle of the planning of a mini-conference at MSRI a few years ago in which he was to be a key participant, but he was still able to participate some by Skype. Against expectations, he recovered enough to do active and impressive research again. He wasn't trying to hide his health challenges but I'm not surprised that it wasn't well known since he was being productive again.

New directions

The deep 'repaying' impact of quantum field theory on mathematics, and in particular in enumerative geometry, has been extraordinary. In the words of S. Katz and D. Cox in [6] (first paragraph of the preface):

> The field of mirror symmetry has exploded onto the mathematical scene in recent years. This is part of an increasing connection between quantum field theory and many branches of mathematics.

Around and after the episode concerning the computation of N_3 (see the first section), there was a stream of important contributions in the direction of mathematical proofs of the pre-

Figure 3. Duco van Straten and Stein A. Strømme during a boat excursion at the 1993 Dyrkolbotn Conference on Mirror Symmetry

dictions of mirror symmetry but the most innovative developments actually came from applications of physical ideas to develop new mathematics. The rest of this section is an attempt to signpost a number of key highlights, with due regard to Strømme's contributions.

1992
M. Kontsevich proves a conjecture of Witten in his PhD thesis *Intersection Theory on the Moduli Space of Curves and the Matrix Airy Function*. It was published as the widely acclaimed paper [42]. Kontsevich is also an invited speaker at the 1st ECM (Paris). The lecture on *Feynman diagrams and low dimensional topology* was published in the conference proceedings (First European Congress of Mathematics, 1992, Paris, Volume II, Progress in Mathematics 120, Birkhauser 1994, 97-121).

S.-S. Yau assembles [67], "the first book of papers published after the phenomenon of mirror symmetry was discovered". The collection includes 20 papers (see AMS for the table of contents), in particular the two landmark contributions *Mirror manifolds and topological field theory* (E. Witten, [62]) and *Topological mirrors and quantum rings* (C. Vafa, [60]). It also includes *Rational curves on Calabi-Yau threefolds* (S. Katz, [37]). The paper [21] was included in the first edition, seemingly without the permission of the authors, but was excluded in the updated version (1998). In a private communication, S. Katz says (19 July, 2014):

> The *Essays on Mirror Manifolds* volume included papers from the Workshop on Mirror Symmetry at MSRI in May 1991. This was the first workshop that brought such a large number of algebraic geometers and string theorists together, and was organised quickly following the work of Stein Arild Strømme/Geir Ellingsrud and Candelas et al. [...] Not many mathematicians have inspired short-notice international conferences at MSRI.

1993
G. Ellingsrud and S. A. Strømme publish [22].

The author of this article made a small contribution [63] to explain the new connections to 'physical mathematics' to the Spanish Differential Geometry community. This was shortly after he attended the 1993 Dyrkolbotn Conference (S. A. Strømme was one of the organisers and the picture with Duco van Straten was taken on that occasion). The papers [21, 22] were cited.

1994
M. Kontsevich and A. Givental plenary lectures *Homological algebra of mirror symmetry* and *Homological geometry and mirror symmetry* at the Zürich ICM were published in the congress proceedings as [44] and [29], respectively. M. Kontsevich also takes part in the conference *Moduli Space of Curves* (Texel Island) and his lecture on *Enumeration of rational curves via torus actions* was included in the proceedings as [43]. In this paper we read:

> The main body of computations is contained in section 3. Our strategy here is quite standard: we reduce our problems to questions concerning Chern classes on a space of rational curves lying in projective spaces (A. Altman - S. Kleiman, S. Katz), and then use Bott's residue formula for the action of the group of diagonal matrices (G. Ellingsrud and S. A. Strømme). As a result we get in all our examples certain sums over trees.

Concerning the work on Bott's formula and enumerative geometry [24] "Kontsevich realised that the method could be applied to the moduli stack of stable maps more easily than it could be applied to the Hilbert scheme of rational curves. This led to the rapid development of Gromov-Witten theory" (Sheldon Katz, private communication, 19 July 2014).

M. Kontsevich and Y. I. Manin publish the landmark paper [45]. V. Batyrev publishes [2]. G. Ellingsrud, J. Le Potier and S. A. Strømme present the work *Some Donaldson invariants of* $\mathbf{CP}^2$ to the Sanda/Kyoto Conference on Moduli of Vector Bundles and it is published in the proceedings as [13].

1995
G. Ellingsrud and S. A. Strømme publish [23] (see the first section of this note). V. V. Batyrev and D. van Straten publish [3]. W. Fulton and R. Pandharipande take part in the Santa Cruz *Algebraic Geometry* Conference and present *Notes on stable maps and quantum cohomology* (included in the proceedings as [28]). D. Morrison and M. Plesser publish [48].

1996
G. Ellingsrud and S. A. Strømme publish [24] (*Bott's formula and enumerative geometry*), for whose influence we refer to the comments under **1994**. A. Givental publishes [30] (*Equivariant Gromov-Witten invariants*). D. Morrison publishes the lecture notes *Mathematical aspects of Mirror Symmetry* (`alg-geom/9609021`). C. Voisin publishes [61], an English translation by R. Cooke of her French notes of the previous year. Publication of [32] (*Mirror Symmetry, II*; see AMS for the table of contents).

1998
M. Kontsevich is awarded the Fields Medal at the ICM-98 (Berlin). Among other contributions, his work on enumerative geometry (and its connection to Witten's conjecture) is cited. G. Ellingsrud and S. A. Strømme publish [25] *An intersection number for the punctual Hilbert scheme of a surface*.

1999
D. Cox and S. Katz publish the comprehensive text [6]. They cite the papers [23, 24] of G. Ellingsrud and S. A. Strømme. Publication of [51] (*Mirror Symmetry, III*; see AMS for the table of contents). Y. I. Manin publishes *Frobenius Manifolds, Quantum Cohomology, and Moduli Spaces* (for more information, see AMS).

2000
Two plenary lectures at the 3rd ECM (Barcelona 2000) are closely related to the topics of this timeline: *The Mathematics of M-Theory* (R. Dijkgraaf) and *Moduli, motives, mirrors* (Y. I. Manin). They were published in the proceedings as [8] and [46]. The latter surveyed the "mathematical developments that took place since M. Kontsevich's report at the Zürich ICM".

2002
Publication of [7] (*Mirror Symmetry, IV*; see AMS for the table of contents).

2003
Publication of the eight-author treatise [35] on *Mirror Symmetry* (*xx*+929 pp. – see AMS for the table of contents). According to Y. I. Manin, it is "a valuable contribution to the continuing intensive collaboration of physicists and mathematicians. It will be of great value to young and mature researchers in both communities interested in this fascinating modern grand unification project."

2005
R. Penrose publishes [50]. Although some critics have issued negative views of different aspects of this purported "complete guide to the laws of nature", it provides many insights to the ways mathematics and physics interact. The section 31.14, for example, is devoted to "The magical Calabi–Yau spaces and *M*-theory".

R. P. Thomas publishes [59].

2006
S. Katz publishes [38], a text based on "fifteen advanced undergraduate lectures I gave at the Park City Mathematics Institute" during the Summer of 2001. He cites the paper [23]. This book, together with the more advanced [6] (for a later stage), is a good starting point for readers wishing to know more about developments in algebraic geometry (and in enumerative geometry in particular) related to string theory. For physical and historical background, references such as [31] and [68] may also be good reading.

2007
J. Kock and I. Vainsencher publish [41]. This lovely text is based on notes in Portuguese prepared "to support a five-lecture mini-course given at the 22° Colóquio Brasileiro de Matemática and published by IMPA in 1999". Two papers of Strømme are cited [55, 57]. The latter paper is also cited in Hartshorne's *Deformation Theory* [34].

2010
M. Atiyah, R. Dijkgraaf and N. Hitchin publish the review *Geometry and Physics* [1] of the "remarkably fruitful interactions between mathematics and quantum physics in the last decades". The authors "point out some general trends" of these interactions and advance ideas about the kind of problems that will have to be solved in order to have a comprehensive understanding in mathematical terms of physical theories involved in a unified view of nature. This paper is included in the seventh volume of Atiyah's Collected Works (Oxford University Press, 2014).

A colleague, leader and fellow human being

On 1 February, Antonio Campillo, President of the Royal Spanish Mathematical Society, suggested that I write an article about Stein Arild Strømme, who had died the day before. After some preliminary contact with colleagues of the small world (Geir Ellingsrud, Sheldon Katz, Ragni Piene, Audun Holme and Steve Kleiman), I wrote to Professor Gunnar Fløystad (a colleague of Sein Arild at the Mathematics Department of the University of Bergen) to find out whether somebody else closer to Strømme had a similar intention. After a few messages, I concluded that I would try to go ahead and so I corresponded with the editors of the EMS Newsletter who very kindly indicated a few guidelines about how to frame such a work. My thanks to all the people named here for their invaluable help.

Professor Gunnar Fløystad, in particular, sent me the texts read at the funeral and the short obituary published in the local newspapers. Since they amount to an institutional appreciation, it is fitting to provide an English translation of the parts that are more relevant here[13] (a few footnotes have been inserted in order to provide information that readers cannot be assumed to know):

> [...] Stein Arild Strømme was born in Oslo in 1951[14] and studied mathematics at the University of Oslo. After his doctorate he came to Bergen in 1984 and soon gained permanent employment at the Department of Mathematics. His specialty was algebraic geometry. He soon gave a strong impression, both by his ability to grasp and solve problems and by his deep understanding and mastery of the field.[15] He was appointed as a professor in 1993.

13 I am grateful to Gunnar Fløystad for helping me polish my rather rough initial translation.

14 His mother Inger Johanne Strømme was a school teacher and his father Sigmund Strømme was a renowned and influential publisher.

15 This can be illustrated with the papers [19] and [17]. They appeared in

In the 1980s and 1990s, he produced significant mathematical articles in collaboration with Geir Ellingsrud, his brother-in-law. In 1988, jointly with Ellingsrud and Professor Christian Peskine of the University of Oslo, he received the NAVF's award for excellence in research. In 1999, he became head of the Mathematics Institute, a position that he held for 11 years. During this period, major changes took place at the institute: there were many new appointments to replace people who had retired and this happened under important changes in the funding and teaching at universities. Stein Arild created the balanced and strong research institute that we have today.

A book has been written about being a head of department. It says you must expect to make one enemy for each year in office. Stein Arild managed the feat of renewing the department while still remaining friends with all his colleagues. He was a member of the science faculty council consisting of deans, the director and other science department heads and he was highly valued and respected in this group.

Stein Arild was a man of quiet manner. He sensed rather than pushed his way through. He kept an online blog and we became worried if too much time passed since the last post. [. . .] He showed concern both for students and for us, his colleagues. When he stepped down as head of the department and the disease seized him, we especially noticed how much he cherished his colleagues. We nevertheless understood when he moved to Oslo and Lambertseter during his last year, to be close to his family.

Our thoughts go out to his wife Leikny and his two sons and their families.[16] Stein Arild has passed away but he has left us with many dear memories that will stay with us. Colleagues and friends at the department and science faculty thank you for what you were to us as a colleague, leader and fellow human being.

Let the distances in space and time be no barriers for the deep expressions of similar feelings of condolence by all his colleagues and friends from so many places around the world.

Remark

In the references below we have included some general treatises that are not cited in the text and which may have been agreeable to Stein Arild: [14], [49], [9], [27], [58], [33], [10], [26] and [11] (forthcoming).

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Inventiones and together they provide an explicit determination of the Chow ring of the Hilbert space of points in the plane. He also worked on vector bundles and moduli spaces: [16, 18, 47, 53, 54, 12].

16 Stein Arild had two sons: Kjetil Strømme, a publisher, with his first wife, and Torstein Strømme, an engineer, with his second wife (Leikny). Kjetil has a daughter, Vanja.

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