***Laudatio* of Professor Sir Michael F. Atiyah on the occasion of his honorary doctoral degree by the Technical University of Catalonia (25 April, 2008)**

By S. XAMBÓ

Rector Magnífic; distingits membres del Claustre i del Consell Social; autoritats acadèmiques, professors, estudiants, personal d'administració i serveis; Presidente, vicepresidente y secretario de la Conferencia de Decanos de Matemáticas; honorable cònsol del Regne Unit a Barcelona; convidats, amigues i amics;
Professor Sir Michael Francis Atiyah:

Bon dia, buenos días, *good morning*, **صباح الخير** (*sabah al-khair*)

The formal protocol of this ceremony calls for "praising the merits of Professor Sir Michael Francis Atiyah". These merits, however, are so clear and extraordinary, and they have been acknowledged by such a numerous list of the highest level instances around the world, that I do not see fit to refer to them here, and even less when you can find a summary in the invitation card [reproduced in the Annex] or, with considerably more detail, in the poster exhibit in this auditorium hall [now available at the author's web site]

Instead I shall make a few comments about his work that have been chosen with the intention of emphasizing some of the virtues of a prodigiously fecund and clear mind. To my understanding, these virtues are the foundation from which have sprung his extraordinary scientific contributions and, as a consequence, the recognitions that he has received.

The first Atiyah waves that reached those of us that were studying mathematics toward the end of the sixties materialized in the form of two books. The title of the first was K-Theory *and was published in 1977 by Benjamin. The second book was*

the "Atiyah-Macdonald"*, which means the* Introduction to Commutative Algebra. *It was published in 1969 by Addison-Wesley. The two books are rather different, and in many respects, but both revealed new ways of creating knowledge. We marvelled at the unsurpassed ability to make that knowledge accessible to us.*

The K-Theory *book is based on the lecture notes for a course taught at Harvard in the fall of 1964. It was ten years since Atiyah had brilliantly finished his bachelor years at Cambridge and in the meantime he had published a stream of articles that unveil the depth and breath of his sight. Among those articles there are those that he wrote in collaboration with Friedrich Hirzebruch on so-called "K theory". Witness of the significance of these works is the fact that in 1962 he delivers the lecture* The Grothendieck ring in geometry and topology *at the International Congress of Mathematicians (ICM) held in Stockholm. The importance of this theory will be even clearer in the subsequent developments related with the "index theory".*

The other book that I mentioned, Introduction to Commutative Algebra*, was written by Michael Atiyah and Ian Macdonald on the occasion of teaching a course at Oxford to third year mathematics students. For many mathematicians it is a model book. It presents to perfection, in less than 140 pages, the essential ideas of the subject, including connections with algebraic geometry and with number theory. The problems at the end of each chapter, chosen with much care, are an integral part of the book design. Solving them systematically is essential for a thorough understanding of the ideas and I would like to say that this task has been considered, since the publication of the book, as a sort of rite of passage for mathematics students. It has been, and still is, a compulsory reference for all mathematics students and for the algebra teachers. The Spanish version, following the translation by Griselda Pascual (1926-2001), was published in 1973 by Reverté and is the version most used by Spanish speaking students.*

Between the 1962 and 1966 ICMs, Michael Atiyah begins research on what Israïl M. Gelfand calls the "index problem" (it refers to the number of independent solutions of elliptic partial differential equations) and soon imagines a strategy to solve it. To carry out this strategy he collaborates with I. Singer (they publish a fundamental paper in 1963) and with Bott (two key papers on this question, one in 1964 and another in 1966). In addition, he generalizes the problem to "manifolds with boundary" (this result appears as an appendix in the volume 57 of the «Annals of Mathematics Studies» of Princeton, *which contains the fruits of a seminar run in 1965 on* The Atiyah-Singer index theorem*).*

> "I realized at the time the significance of the index theorem and that it represented the high point of my work, but it would have been hard to predict that the subject would continue to occupy me in various forms for the next twenty years. I would also have been extremely surprised if I had been told that this work would in due course become important in theoretical physics" (Commentary to the third volume of Atiyah's *Collected Works*).

We come to the 1966 ICM (held in Moscow) in which M. Atiyah is deservedly awarded the Fields medal:

> *Michael Francis Atiyah (Oxford University).* Did joint work with Hirzebruch in K-theory; proved jointly with Singer the index theorem of elliptic operators on complex manifolds; worked in collaboration with Bott to prove a fixed point theorem related to the "Lefschetz formula".

It is an obligation to say here a few words on the memoir Elliptic operators and compact groups (Springer-Verlag, 1974). *It is based on the course of lectures taught by Atiyah in 1971 at the Institute for Advanced Study and in some ways it is related to the Atiyah-Singer index theory as the* K-Theory *book was related to the K theory of Atiyah-Hirzebruch. It is perhaps less well known, but it is masterful in its*

purpose (an extension of the index theory to operators "transversally elliptic"), in the variety and sophistication of the techniques used, and in the brevity (less than hundred pages).

The following Atiyah wave reached us at the end of the seventies. More concretely, in 1979, with the memoir The Geometry of Yang-Mills fields*, corresponding to the "Lezioni Fermiane" sponsored by the Accademia Nazionale dei Lincei and the Scuola Normale Superiore de Pisa. At the time of giving these lectures, it is not more than one year from the moment in which Atiyah has oriented his interests towards the so-called "gauge theories", or "Yang-Mills" theories, with the attention focussed on the interaction between geometry and physics. Delivered to a mixed audience of physicists and mathematicians, it brings about a synthesis of several points of view, including Penrose's "twistor programme" approach, and it contains constructions and results much celebrated then, as for example the construction of self-dual solutions of the Yang-Mills equations (also called "instantons") due to Atiyah, Ward, Drinfeld and Manin. As in the memoirs referred to before, it has less than one hundred pages.*

The influence of this memoir, and of the numerous works advancing in similar directions that followed it, has been immense. No doubt it can be taken as the point in which the constructive interaction between physics and mathematics was restored after its fading away in the first half of the XX century (with some exceptions, as in the case of Hermann Weyl). In fact this restoration came about with a bonus, for if until then it was the mathematics that were applied to physics, it became ever more evident that the ideas inspired in physics could have extraordinary consequences in the world of mathematics.

The crest of the singular wave that followed can be identified with the publication in 1990 of the brief memoir The Geometry and Physics of Knots*, which assembles the lectures delivered in Florence by Atiyah in 1988 sponsored by the Accademia Nazionale dei Lincei. The focus of this wonderful work is to explain "the Jones theory, transformed by Witten into a Topological Quantum Field Theory". This is a*

truthful statement, but to understand its scope it is necessary that we look back at three developments that took place between The Geometry of the Yang-Mills Fields *and* The Geometry and Physics of Knots*, while keeping an eye on the role played by Atiyah in those events.*

A first rather surprising development, occurred at the beginning of the eighties, was the work of Simon Donaldson, a student of Atiyah. In this work he used the Yang-Mills equations to deduce spectacular results on the geometry in dimension 4 (the dimension of Einstein's space-time). For these works, Donaldson was awarded the Fields medal in the 1986 ICM (Berkeley).

The second development was associated with the physicist Edward Witten, who deserves a very special mention. Said in Atiyah's measured words, "[…] his command of mathematics is rivalled by few mathematicians, and his ability to interpret physical ideas in mathematical form is quite unique. Time and again he has surprised the mathematical community by a brilliant application of physical insight leading to new and deep mathematical theorems". These words can be found in Atiyah's description of Witten's work at the Kyoto ICM (1990), where he was awarded the Fields medal. He is thus the only physicist having received this distinction.

The third development that has to be considered is Vaughan Jones' work. He obtained, in the late eighties, new invariants of knots (objects of a purely topological nature) using ideas related to physics. For these works Jones was also awarded the Fields medal at the Kyoto ICM.

It is at this point that Atiyah presents a paper at the Hermann Weyl symposium (1987) in which he conjectures that Donaldson's theory and Jones' invariants can be explained using quantum field theory. This interpretation is published soon after by Witten in two articles published in 1989, one on Jones and the other on Donaldson, that initiate topological quantum field theory. In this way we gain a better understanding of the purpose of The Geometry and Physics of Knots*: "to explain the Jones theory, transformed by Witten into a TQFT".*

*I have spoken about a very significant part of Michael Atiyah's work, but I have to say that in extension it represents only a small fraction of his production. Indeed, his six volumes of "Collected Works" , published in 1988 (5 volumes) and 2004 (1 volume), assemble more than four thousand pages, while the memoirs about which I have talked (*K-Theory, The Geometry of Yang-Mills Fields, Elliptic operators and compact groups, Classical groups and classical differential operators on manifolds, and The Geometry and Physics of Knots*) add up to less than five hundred pages.*

It is a pleasure to recall here that Michael Atiyah was awarded in 2004, jointly with Isadore Singer, the Abel Prize "for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics".

An interesting aspect of Atiyah's Opera *is that it contains a good number of pieces (nearly forty), qualified as "quasi-mathematical" by the author, that consist in essays, surveys or biographical writings. In extension they occupy less than one tenth of the Collected Works, but their importance is great because they offer more leisurely ways to access Atiyah's thinking than the more technical articles. In addition, they reveal that Atiyah is also a first rate writer. His prose is fluent, sharp, spotless, effective. It trips over nothing, does not contain unnecessary things and it always gives the impression that it has said all that was to be said.*

*Michael Atiyah visited Barcelona in 1997, in 1998 and in December 2007. The purpose of the 1997 and 1998 visits was to chair meetings of the Scientific Committee of the 3rd European Congress of Mathematics (3**ecm**). On those occasions the host institution was the Societat Catalana de Matemàtiques of the Institut d'Estudis Catalans, responsible of the organization of the 3ecm by delegation of the European Mathematical Society. Let me recall here that the sustained efforts of Michael Atiyah played a decisive role in its foundation.*

The host institution for the 2007 visit was the Facultat de Matemàtiques i Estadística (FME) of the Universitat Politècnica de Catalunya and the purpose was

to deliver the keynote lecture Riemann's influence in Analysis, Geometry and Number theory *of the FME Riemann year.*

In the December 2007 visit, Atiyah was also invited by the Centre de Recerca Matemàtica to deliver a lecture on Duality in Mathematics and Physics. This lecture was delivered at the Facultat de Matemàtiques of the Universitat de Barcelona.

Let me go back for a moment to the World Mathematical Year. One of the many activities of Michael Atiyah on that occasion was chairing the International Mathematical Union project that turned out to be called Mathematics: Frontiers and Perspectives *(American Mathematical Society, 2000). One of the papers, by W. T. Gowers (*The two cultures of Mathematics*), includes the following quotation of the 1984 interview by Minio:*

> ***Minio.*** *How do you select a problem to study?*
>
> ***Atiyah.*** *I think that presupposes an answer. I don't think that's the way I work at all. Some people may sit back and say, "How do I solve this problem?" I don't. I just move around in the mathematical waters, thinking about things, being curious, interested, talking to people, stirring up ideas; things emerge and I follow them up. Or I see something which connects up with something else I know about, and I try to put things together and things develop. I have practically never started off with any idea of what I am going to be doing or where it's going to go. I'm interested in mathematics; I talk, I learn, I discuss and then interesting questions simply emerge. I have never started off with a particular goal, except the goal of understanding mathematics.*

Professor Sir Michael Francis Atiyah, we are very grateful, very happy, and very proud of your having accepted our invitation to receive an honorary degree from our university, and, by extension, from our mathematical community at large.

Let me end by trying to welcome you into our institution, into our mathematical community, in one of the languages that is dear to you from your childhood and youth:

(*marhában bik fi jamatina*) **مرحبا بك في جامعتنا**

Annex: Text included in the invitation card to the ceremony

Michael Frances Atiyah (London, 1929) obtained his university degrees from Cambridge (UK). Student of W. Hodge, he was elected Fellow of Trinity College (Cambridge) in 1954 and one year later obtained his doctorate. Soon after he went to the Institute for Advanced Study (IAS) in Princeton, then to Cambridge (lecturer in 1957, Fellow of Pembroke College in 1958), and then to Oxford (1961), where he was a Fellow of St Catherine's College. In the period 1963-1969 he held the Savilian Chair of Geometry at Oxford. After an appointment in 1969 as Professor of Mathematics at the IAS, he became a Royal Society Research Professor at Oxford in 1971 and a Fellow of St Catherine's College. He remained in this position until 1990, when he became Master of Trinity College (Cambridge) and the first Director of the Isaac Newton Institute for Mathematical Sciences. President of the Royal Society (1990-1995) and Chancellor of the University of Leicester (1995-2007), at present he is an Honorary Professor at the University of Edinburgh and President of the Royal Society of Edinburgh. In the year 2000, Professor Atiyah chaired the Programme Committee of the 3ecm of the EMS (Barcelona).

The scientific achievements of Professor Atiyah up to 2004 are faithfully mirrored in the six volumes of his Collected Works (CW). Published by OUP, with a total of more than four thousand pages, they constitute an invaluable treasure of results and ideas that will keep inspiring the newer generations as it has been the case for the last decades. They are the *de rigueur* source for the original breakthroughs and developments that are the landmark of a great mind, a fact that has been acknowledged by an impressive list of distinctions, including the highest that are awarded in the field of mathematics: the Fields Medal (1966) and the Abel Prize (2004), the latter shared with I. Singer for a result (the Atiyah-Singer index theorem) that uncovered deep connections between geometry and analysis (CW 3 and 4) and later between these fields and physics (CW 5). Felicitously, he is foreign member of the Royal Academy of Sciences of Spain.

The collaboration with Singer persisted for over twenty years. Two of the other many scientific partnerships of Atiyah were as intense and fruitful as the one with Singer: with F. Hirzebruch, for more than a decade, on the foundations of topological K-theory and its use in solving many difficult outstanding problems (CW 2), and with R. Bott, for over twenty years, especially remembered by the Atiyah-Bott fixed point formula. In the late 70's, Atiyah began to

built strong new bridges between mathematics and theoretical physics affording a stream of new ideas and fresh energy for an unprecedented flourishing of both (CW 5 and 6). Especial mention here is deserved by his collaboration with E. Witten, since the early 90's, that has led to deep insights into the intricate and perplexing nature of the two-way relationship between mathematics and quantum field theory.

Professor Atiyah has had many students (S. Donaldson, N. Hitchin, F. Kirwan, P. Kronheimer, G. Lusztig, G. Segal, to name a few) and is fully active in research and other commitments. Currently, for example, he is cooperating with the distinguished neurophysiologist S. Zeki in studies of the human brain, particularly when it carries out mathematical tasks.