

Landau notation

In order to deal with results in extremal combinatorics we need to recall the so-called Landau notation (also called asymptotic notation):

Def/ Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be functions. Then we write that $f(x) = O(g(x))$ if $|f(x)| \leq C|g(x)|$ for all $x \geq x_0$. On the other direction, if $f(x) = O(g(x))$ then $g(x) = \Omega(f(x))$.

Def/ Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be functions. Then we write that $f(x) = o(g(x))$ if $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$. Equivalently, if $f(x) = o(g(x))$, then $g(x) = \omega(f(x))$.

We finally write $f(x) = \Theta(g(x))$ if $f(x) = O(g(x))$ and $g(x) = O(f(x))$. There is some equivalence notation used more frequently in other disciplines (see, analytic number theory):

Vinogradov's not.	Landau's notation
$f(x) \ll g(x)$	$f(x) = O(g(x))$
$f(x) \gg g(x)$	$f(x) = \Omega(g(x))$
$f(x) \sim g(x)$	$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1$
$f(x) \asymp g(x)$	$f(x) = \Theta(g(x))$

So, in particular, if $f(x) = g(x) + o(g(x))$, then $f(x) \sim g(x)$.

