Problem Sheet 4

Structural results for (1-,2-,3-) connected graphs. Menger's Theorem

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Discrete Mathematics II, Winter 2013-2014
Deadline: 19th November 2013 (Tuesday) by 10:00, at the end of the lecture.

Problem 1: Let G be a graph. Prove that every vertex of G has even degree if and only if every block of G is Eulerian.

Problem 2: Let G be a graph without isolated vertices. Prove that G has no even cycles, then every block of G is either an edge or an odd cycle.

Problem 3: A connected multigraph is *series-parallel* if it can be obtained from a tree by means of the following two operations: subdividing an edge (series extension), and duplicating an edge (parallel extension). A connected graph is *outerplanar* if it is planar and in can be drawn in the sphere in such a way that all vertices are adjacent with a unique face.

1.- Characterize 2-connected series-parallel graphs and 2-connected outerplanar graphs (namely, how can you construct them?).

2.- Show that series-parallel graphs exclude K_4 as a minor. Show that outerplanar graphs exclude K_4 and $K_{2,3}$ as minors.

3.- Show that outerplanar graphs are series-parallel.

4.- Show that there are not 3-connected series-parallel graphs. (*Hint*: you can apply Tutte's Theorem for the structure of 3-connected graphs).

(Note: in fact, series-parallel graphs are exactly $\mathrm{Ex}(K_4)$, and outerplanar graphs are $\mathrm{Ex}(K_4, K_{2,3})$, but this requires a little more of work...)

Problem 4: Apply the edge global version of Menger's Theorem to prove that if G is a cubic graph, then $\kappa(G) = \kappa'(G)$.

Problem 5: Prove König's Theorem for the maximum size of a matching in a bipartite graph using the vertex local version of Menger's Theorem.