Problem Sheet 3

Matching in general graphs. Connectivity

Juanjo Rué
Discrete Mathematics II, Winter 2013-2014
Deadline: 12th November 2013 (Tuesday) by 10:00, at the end of the lecture.

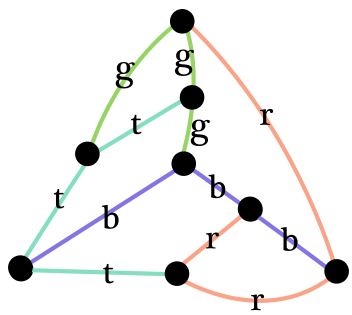
Problem 1: Prove that

1.- every tree has at most one perfect matching.

2.- a tree T has a perfect matching if and only if $o(T - v) = 1$ for every vertex of T.

Problem 2: Prove Hall's Theorem from Tutte's Theorem. (*Hint:* show that if a bipartite graph does not satisfy Hall's condition, then it does not satisfy Tutte's condition).

Problem 3: Let G be a bridgeless cubic graph. Prove that we can cover G with paths of length four. Here you have an example for a concrete graph:



(*Hint*: apply Petersen's Theorem, and study the structure of the complement of the perfect matching in G).

Problem 4: Take $0 < r \leq s \leq t$ positive integers. Construct a graph G with $\kappa(G) = r$, $\kappa'(G) = s$ and $\delta(G) = t$.

Problem 5: Let G be an r-connected graph with an even number of vertices without $K_{1,r+1}$ as an induced subgraph. Prove that G has a 1-factor (*Hint:* verify Tutte's condition: for a set $S \subseteq V(G)$ distinguish the easier cases $0 \leq |S| < r$ and for the more involved situation $|S| \geq r$ prove that, in fact, the total number of components of $G[V - S]$ (not necessary odd) is smaller than $|S|$).