Problem Sheet 2

Graph minors. Matchings and 1-factors in bipartite graphs

Juanjo Rué

Discrete Mathematics II, Winter 2013-2014

Deadline: 5th November 2013 (Tuesday) by 10:00, at the end of the lecture.

Problem 1: Prove the equivalence between the statements of Robertson-Seymour Theorem:

1.- A minor-closed family is characterized by a *finite* set of excluded minors.

2.- The set of graphs is well-quasi ordered with respect to the minor relation.

Problem 2: Prove that trees are *not* well-quasi ordered by the subgraph relation. Prove the same for the induced subgraph relation.

Problem 3: For each $k > 1$, construct a k-regular simple graph without 1-factors.

Problem 4: Let $\mathbf{A} = (A_1, \ldots, A_m)$ be a collection of subsets of a set Y. A *system of distinct representatives* for $\mathbf{A}$ is a set of distinct elements $a_1, \ldots, a_m$ in Y such that $a_i \in A_i$. Prove that $\mathbf{A}$ has a system of distinct representatives if and only if $\left|\bigcup_{i \in S} A_i\right| \geq |S|$ for every $S \subseteq [m]$.