Problem Sheet 13

The Second Moment Method and Chebyshev's Inequality

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Discrete Mathematics II, Winter 2013-2014

Deadline: 11th February 2014 (Tuesday) by 10:00, at the end of the lecture.

Problem 1: Show that Chebyshev's inequality cannot be improved. In order to do so, Consider the random variable X which takes values $\{-1, 0, 1\}$ with probabilites $\mathbb{P}(X = -1) = \mathbb{P}(X = 1) = \frac{1}{2a^2}$, $\mathbb{P}(X = 0) = 1 - \frac{1}{a^2}$.

Problem 2: mimic the proof done in class in order to show that the function $r(n) = n^{-2/3}$ is a threshold function for p concerning the property '$\mathcal{G}(n, p)$ has K_4 as a subgraph'.

Problem 3: Consider the probability space where Ω is the set of subsets of $[n]$, and each element in $[n]$ is in the random set with probability $p := p(n) = n^{-\alpha}$, for a certain positive value of α. Show that $r(n) = n^{-2/3}$ is a threshold function for p concerning the property 'a random subset of $[n]$ contains a 3-AP'.