

Problem Sheet 0

Warm-up

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These problems will be corrected the 22nd October 2013 during the problem session.

Problem 1: Every connected graph has an spanning tree.

Problem 2: A connected graph is Eulerian if and only if every vertex has even degree.

Problem 3: The diameter of a connected graph G ($\text{diam}(G)$) is the greatest distance between two vertices in G . A vertex is *central* in G if its greatest distance from any other vertex is as small as possible. This distance is the radius of the graph ($\text{rad}(G)$).

Prove that:

$$\text{rad}(G) \leq \text{diam}(G) \leq 2\text{rad}(G).$$

Problem 4: Prove that

- 1.- every tree is bipartite.
- 2.- every tree has at least 2 vertices of degree 1.
- 3.- there exist one vertex of degree 1 on its largest partite set.

Problem 5: Let $k > 1$. Prove that $2k$ -regular graphs do not have bridges. Construct, for each $k > 1$, a $(2k + 1)$ -regular graph with some bridge.

Problem 6: Let P and Q be paths of maximum length on a connected graph G . Prove that P and Q have a common vertex.