Mock Exam

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Discrete Mathematics II, Winter 2013-2014

Deadline: 14th January 2014 (Tuesday) by 10:00, at the end of the lecture.

Problem 1 (2 points):

1. State the definition of perfect graph (0.5 points).
2. Prove that the complement of a bipartite graph is perfect (1.5 points).

Problem 2 (2 points):

1. State Turán's Theorem (0.25 points) and Erdős-Stone Theorem (0.25 points).
2. State Erdős-Simonovits Corollary (0.5 points) and prove it by using Turán's Theorem and Erdős-Stone Theorem (1 point).

(Note: You do NOT have to prove neither Turán's Theorem nor Erdős-Stone Theorem).

Problem 3 (2 points): Prove that if the graph G has a Hamiltonian cycle (namely, a connected 2-factor), then G has a 4-flow.

Problem 4 (2 points): Prove that the Petersen graph is not planar.

Problem 5 (2 points): Let M be a planar map defined from a simple graph G, where all faces have length 3.

- Prove that the corresponding dual map M^* has not bridges (0.25 points).
- Prove that the corresponding dual map M^* has a 2-factor (0.5 points).
- Show that the vertices of the initial map M can be coloured using 2 colours (NOT a proper colouring) in such a way that every face is incident with vertices of both colours (1.25 points).

(Note: you can solve each part independently).

- All problems will will be properly graded, but will NOT be counted towards your homework requirements.
- You may choose to time yourself (2 hours will be the time you will have for the Final Exam).
- You should try to write ALL the steps in detail.
- The 15th January a precise correction of the exam (with all the precise grading in each problem) will be available on the web.