Problem Sheet 11

Markov chains

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Deadline: 7th July 2014 (Tuesday) by 10:00, at the end of the lecture.

`Problem 1 [10 points]:` A die is rolled repeatedly. Which of the following are Markov chains? For those which are homogeneus, give the transition matrix:

- The largest number X_n shown in the n-th roll.
- The number N_n of sixes in n rolls.
- At time r, the time C_r since the most recent six.

`Problem 2 [10 points]:` Prove that

1. $P_{ii}(s) = 1 + F_{ii}(s)P_{ii}(s)$.
2. $P_{ij}(s) = F_{ij}(s)P_{jj}(s)$, if $i \neq j$.

Prove also that if three states i, j, k satisfy that $i \leftrightarrow j$, $j \leftrightarrow k$, then $i \leftrightarrow k$.

`Problem 3 [10 points]:` Let $\{X_n\}_{n\geq 1}$ a sequence of independent identically distributed random variables, and write $S_n = \sum_{r=1}^{n} X_n$, $S_0 = 0$, $Y_n = X_n + X_{n-1}$ (assuming that $X_0 = 0$) and $Z_n = \sum_{r=0}^{n} S_n$. Which sequence ($\{S_n\}$, $\{Y_n\}$, $\{Z_n\}$) defines a Markov chain?

`Problem 4 [10 points]:` Let $\mathbf{X} = \{X_n\}_{n\geq 0}$ be a Markov chain with an absorving state s such that all the other states i communicate. Show that all states other than s are transient.