

Make-Up Exam

Discrete Mathematics III, Winter 2013-2014
Jun. Prof. Juanjo Rué

Problem 1 (2 points):

- Define combinatorial class, admissible combinatorial class and generating functions associated to an admissible combinatorial class (0.5 points).
- Define and get the generating function associated to a multiset of a given combinatorial (1.5 points).

Problem 2 (2 points):

- Define properly rooted planar map. (0.5 point).
- Justify that there is a bijection between rooted planar maps with n edges and rooted planar quadrangulations with n faces (1.5 point).

Problem 3 (2 points): Study the singular expansion of the following combinatorial families:

- The EGF for derangements: $D(z) = \frac{e^{-z}}{1-z}$.
- The EGF for permutations without cycles of length 1 or 2: $E(z) = \frac{e^{-z-z^2/2}}{1-z}$.

Generalize this results by obtaining asymptotic estimates for the number of permutations without cycles of lengths in a set $\Delta \in \mathbb{N}$.

Problem 4 (2 points): Consider the combinatorial class of words over the alphabet $\{0,1\}$, where the size of a word is the number of letters, and with the restriction that the words do not have the pattern 11. Find the GF associated to this combinatorial class. Using the extra variable u , refine the previous GF in order to get a bivariate GF encoding also the number of 1's.

Problem 5 (2 points): *A family of unrooted colored trees:* Consider the family \mathcal{T} of unrooted non-embedded labelled trees where are of two colours: red and blue (R and B for now on). Write $T(x)$ the EGF of this family, where the size of a tree is the number of vertices. The only restriction we assume is that two vertices of type R are *not* incident.

Consider the auxiliary family of rooted trees at a vertex of type R and B , $T_R(x)$ and $T_B(x)$ respectively.

- Find a system of 2 equations defining $T_R(x)$ and $T_B(x)$. Get a single equation defining $T(x)$. (1 point)
- Apply the Dissymmetry Theorem for trees in this family, and get an expression of $T(x)$ in terms of (vertex and edge)-rooted families of trees (0.5 points).
- Find a closed expression of $T(x)$ in terms of $T_R(x)$ (0.5 points).

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- You should try to write and justify ALL steps.
 - The grading of the subject will be available the 23th September (afternoon) both in the webpage and in my office.
 - You can come to my office on Wednesday 24th September from 09:00 to 11:00 to see the exam.