Probability and Random Processes
2013-2014. Fall term

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Objectives

To provide general theoretical results as well as mathematical tools suitable for modelling random phenomena.

Study of specific applications of the theoretical concepts.

- Transform methods: generating and characteristic functions.
- Stochastic convergence problems: types of convergence, law of large numbers, central limit theorem.
- Random processes: branching processes, random walks, Markov chains, Poisson process.

Contents

1. Generating Functions and Characteristic Function (6 h.)

   Probability and moment generating functions. The characteristic function. Convolution theorem. Joint characteristic function of several random variables.

   Applications: Sample mean and sample variance. Sum of a random number of independent random variables. Distributions with random parameters.

Contents

2. Branching Processes (3 h.)

   The Galton-Watson process. Application to population growth. Probability of ultimate extinction. Probability generating function of the n-th generation.

Contents

3. The Multivariate Gaussian Distribution (3 h.)

   Joint characteristic function of independent Gaussian random variables. The multidimensional Gaussian law. Linear transformations. Linear dependence and singular Gaussian distributions. Multidimensional Gaussian density.

Contents

4. Sequences of Random Variables (4,5 h.)

   The weak law of large numbers and convergence in probability. The central limit theorem and convergence in distribution. Mean-square convergence. The strong law of large numbers and almost-sure convergence.

   Applications: Borel Cantelli lemmas. Examples of application.

Contents

5. Stochastic Processes: General Concepts (4,5 h)

   The concept of a stochastic process. Distribution and density functions of a process. Mean and autocorrelation. Stationary processes. Ergodic processes.

Contents

6. Random Walks (4,5 h)

   One-dimensional random walks. Returns to the origin. The reflection principle. Random walks in the plane and the space.

7. Markov chains (7,5 h)

   Finite discrete time Markov chains. Chapman-Kolmogorov equations. Chains with absorbing states. Regular chains. Stationary and limitting distributions.

   Applications: The gambler's ruin problem. Montecarlo methods.

Contents

8. The Poisson process. (6 h.)

   The Poisson process. Intertransition times. Birth and death processes. Continuous time Markov chains.

   Applications: Basic concepts of queueing theory.

Prior skills

- Elementary probability calculations.
- Basic probability models: binomial, geometric, Poisson, uniform, exponential and normal distributions.
- Random variables. Joint probability distribution and density functions. Conditional expectations.
- Elementary matrix algebra. Derivation and integration of functions. Power series.

Bibliography

Basic:

- Gut, A.; An Intermediate Course on Probability. Springer Verlag, 1995.
- Durret, R.; Essentials of Stochastic Processes. Springer-Verlag, 1999.

Bibliography

Complementary:

- Tuckwell, H.C.; Elementary Applications of Probability Theory. Chapmand & Hall, 1995.
- Sanz Solé, M.; Probabilitats. Univ. de Barcelona, 1999.
- Ross, S.M.; Introduction to Probability Models, Academic Press, 2006.
- Grimmet, G.R.; Stirzaker, R.R.; Probability and Random Processes. Oxford Univ. Press, 2001.
- Grinstead, C.M.; Snell, J.L; Introduction to Probability. AMS. `http://www.dartmouth.edu/~chance/teaching_aids/books_articles/probability_book/book.html`

Qualification

- Midterm exam: 5 November 2013
- Final exam: 9 January 2014

Final grade (NF):

$$NF = \max(EF, 0.4\,EF + 0.4\,EP + 0.2\,T)$$

where EF is the final exam mark, EP is the midterm exam mark, and T is the mark of the exercises and assigned work throughout the course.