GDHAM15

Global Dynamics in Hamiltonian Systems

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Abstracts Book

Contents

Invited Talks

Numerical computation of invariant objects with wavelets

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In certain classes of dynamical systems invariant sets with a strange geometry appear. For example the iteration of two-dimensional quasi-periodically forced skew products, under certain conditions, gives us Strange Non-Chaotic Attractors.

To obtain analytical approximation of these objects it seems more natural to use wavelets instead of the more usual "fourier approach". The aim of the talk is to describe the algoritm for the semi-analitical computation of the invariant object (numerical computations of the wavelet coefficients) using both daubechies and Haar wavelets.

The aim for this exercise is twofold. From one side to be able to study bifurcations and "pinching" of the object and from another side to get estimates of the regularity of the object. The sudy of this regularity depending on parameters may give another point of view to the "fractalization routes" described in the literature and that are currently under discussion.

The Fermi-Pasta-Ulam problem: old ideas, recent results, open problems

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In 1954 Fermi, Pasta and Ulam for the first time used a computer to understand the ergodic behavior of a dynamical system with many degrees of freedom, interesting to investigate the very foundations of Statistical Mechanics. Several branches of physical and mathematical investigations started from that paper. The aim of the talk is to revisit, in the light of some recent numerical results, some significant ideas and conjectures on the model. In particular:

(i) The presence, in the model, of (at least) two well separated time-scales: a short one, where only a few normal coordinates share energy, and a larger one, where energy equipartition among all normal modes occurs and the behavior of the model, in view of Statistical Mechanics, is regular.

(ii) The fact that in the short time scale the dynamics of FPU, in spite of the partial energy sharing, is essentially integrable and closely follows the dynamics of the Toda model, while in the large time scale nonintegrability becomes manifest.

The stability of results in the limit of large N (ideally, the search of uniformity in N) will play a central role. The gap between numerical insights and mathematical results, unfortunately, is large.

Billiards in the three body problem

Sergey Bolotin

Moscow Steklov Mathematical Institute and University of Wisconsin

We consider the plane three body problem with two of the masses much smaller than the third one. Periodic solutions with near collisions of the small bodies were named by Poincaré second species periodic solutions. The description of such solutions is reduced to a billiard type system with the discrete Lagrangian determined by the classical Lambert's problem. In the limit of many revolutions between near collisions the billiard system admits relatively simple description.

Classification of constrained differential equations embedded in the theory of slow fast systems

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Modeling real life phenomena such as the heartbeat, the electrical activity of neuron cells, chemical reactions, turbulent flows, or the weather forecast, often involves multi-scale data, therefore they may be modeled by slow fast systems (SFSs). These are ordinary differential equations which depend on a small parameter in a singular way. When such a small parameter is set to zero, one obtains a constrained differential equation (CDE).

This talk will treat recent results regarding the geometric theory of constrained differential equations, slow fast systems, and their relationship. In particular, an extension to the Takens's classification of singularities of CDEs and a unified way to study a large class of SFSs will be presented.

Arnold diffusion in the planar elliptic restricted three-body problem

Maciej Capiński

(in collaboration with Marian Gidea and Rafael de la Llave)

AGH University of Science and Technology, Krakow, Poland

We present a diffusion mechanism for time dependent perturbations of autonomous Hamiltonian systems. The mechanism is based on shadowing of pseudo-orbits generated by two dynamics: an outer dynamics, given by homoclinic trajectories to a normally hyperbolic invariant manifold, and an inner dynamics, given by the restriction to that manifold. On the inner dynamics the only assumption is that the system preserves area. Unlike other approaches, we do not rely on the KAM theory and/or Aubry-Mather theory; we also do not need to check twist conditions nor non-degeneracy assumptions near resonances. As an application, we study the Jupiter/Sun system restricted three-body problem. We show that for any positive and sufficiently small value of the eccentricity of the main body, there are orbits that connect the Lyapunov orbits for different energies in such a way that the change of energy is bigger than a fixed number.

Stability of some dissipative systems and efficient estimates on the existence of quasi-periodic attractors

Alessandra Celletti
(in collaboration with R. Calleja and R. de la Llave)
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The talk concerns recent results about the stability of some dissipative systems, precisely conformally symplectic systems. We describe a suitable KAM theory, which allows to prove the persistence of invariant attractors.

The proof is constructive and it provides efficient algorithms to evaluate the breakdown threshold of quasi-periodic attractors. Applications to model problems are provided.

Transition Chain of Incomplete Intersection

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In the celebrated example of Arnold [1], the diffusion orbits are constructed by so-called transition chain, namely, the stable manifold of each invariant circle intersects its unstable manifold transversally. A variational version was formulated in [2]. If the intersection is not transversal, one can still construct connecting orbits if some restrictions are imposed on the diffusion path. We shall formulate it and show its application in the study of Arnold diffusion in nearly integrable Hamiltonian systems with multiple degrees of freedom.

[1] V.I. Arnol'd, Instability of dynamical systems with several degrees of freedom, (Russian, English) *Sov. Math., Dokl.*, **5**(1964), 581–585; translation from *Dokl. Akad. Nauk SSSR*, **156**(1964) 9–12.

[2] C.-Q. Cheng and J. Yan, Existence of diffusion orbits in a priori unstable Hamiltonian systems, *J. Differential Geometry*, **67** (2004) 457–517.

The measure of invariant tori in nearly-integrable mechanical systems

Luigi Chierchia

Università degli Studi Roma Tre, Roma, Italy

Standard KAM theory predicts that the measure of the density of the union of (Lagrangian, Diophantine) invariant tori in real-analytic nearly-integrable Hamiltonian systems, with perturbation parameter ϵ, is $1-\sqrt{\epsilon}$ (as $\epsilon \to 0$).

There are good reasons to believe that the "right" density, in general, is $1-\epsilon$ (compare Arnold, Kozlov and Neishtadt, Springer Encyclopedia). We prove that such density is at least $1-\epsilon\log(\epsilon)$ for mechanical systems (i.e., systems with Hamiltonian $|y|^2/2+\epsilon f(x)$, (y,x) being standard action-angle variables), provided f belongs to a suitable general (and generic) set.

Generic Mañé Sets

Gonzalo Contreras

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Let M be a compact two-dimensional surface and L a Tonelli Lagrangian on TM. We prove that there is an open and dense set of C^2 real functions $f : M \to \mathbb{R}$ such that the Mañé set of the Lagrangian $L+f$ is a hyperbolic periodic orbit or fixed point.

Quasi-periodic equilibria in quasi-periodic media

Rafael de la Llave
(in collaboration with X. Su and L. Zhang)

School of Math., Georgia Inst. of Technology, Atlanta, USA & École Normale Supérieure, Paris, France

We consider models from statistical mechanics on quasi-periodic media. An example is the quasi-periodic Frenkel-Kontorova model given by the energy

$$\sum_j (u_{j+1} - u_j)^2 + V(\alpha u_j),$$

where $u_i \in \mathbb{R}$, $V : \mathbb{T}^d \to \mathbb{R}$, and $\alpha \in \mathbb{R}^d$.

We are interested in finding critical points, that is solutions of

$$u_{i+1} + u_{i+1} - 2u_i + (\alpha \cdot \nabla)V(\alpha u_i) = 0.$$

Specially, when these solutions are minimizers.

We are also interested in "depinning phenomena", that is the persistence of equilibria when we add an external force.

In the case that $d = 1$, there are two very satisfactory theories: the Aubry-Mather theory studying minimizers and the KAM theory.

In the quasi-periodic case, there are some new developments. P.L. Lions and Souganidis gave simple examples when $d \geq 2$ with no plane-like minimizer as it is well known from Aubry-Mather theory (one should also recall examples of H. Federer in quasi-periodic geodesic flows). Other features of Aubry Mather theory were shown to persist by Gambadudo, Petit, Thieulen by topological methods.

Using KAM theory, we show the existence of open sets of models with quasi-periodic solutions which are plane like. In the case that the models are ferromagnetic (e.g. the FK models), the solutions are minimizers. The models considered are significantly more general than the FK model since they allow multi-body and long range interactions.

The quasi-periodic solutions produced are of two types: with a non-resonant frequency and those with a resonant one. The first ones are destroyed with the addition of an external force, while the second ones persist under addition of forces. Note that in the $d = 1$ case, there are generically no continuous families of generic minimizers while, when $d \geq 2$ we show they exist and are stable.

The method of proof is a KAM scheme based on adding extra equations that allow the solution of the linearized equilibrium. Note that the problems do not admit a transformation theory.

The result has an a-posteriori format and hence leads to several consequences (some almost-periodic models, smooth dependence on parameters, Whitney dependence on the frequencies). The method of proof leads to very efficient algorithms, some of which have been implemented, leading to quantitative conjectures about the phenomena that happen at the breakdown of KAM theory and of Aubry-Mather theory and of the depinning transition.

There are several open problems.

The entropy spectrum of Lyapunov exponents in non-hyperbolic skew-products

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We consider transitive skew-products dynamics containing (and thus mixing) hyperbolic sets of different indices. Jointly motivated by the examples of robustly transitive diffeomorphisms, dynamics arising in the unfolding of heterodimensional cycles and porcupine-like horseshoes, we identify some abstract properties that enable us to describe the topological entropy of level sets for central Lyapunov exponents.

One of the ingredients is a restricted variational principle to describe the topological entropy of certain level sets taking in consideration measures in a certain subset of hyperbolic measures only.

Stable manifolds of parabolic points through the parameterization method

Ernest Fontich
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We consider n dimensional maps with a fixed point whose linearization is the identity (parabolic points). We introduce some quantities related to the relevant non linear terms and we study the existence, regularity and dependence of parameters of stable sets in terms of the regularity of the map and the introduced quantities. In general the stable sets are manifolds which are less regular than the map at the fixed point, but in some cases there is also a loss of regularity at other points.

For that we use the parameterization method which first gives a suitable approximation and then provides the true invariant set. We provide an algorithm for obtaining the approximations which in some cases requires solving an auxiliary partial differential equation. When the stable sets are of dimension two or higher the approximations may not be polynomial, but sums of homogeneous functions.

The case of parabolic fixed points for differential equations is also considered. As an application we consider the manifolds of the infinity of the elliptic spatial RTBP.

Classical statistical mechanics of realistic models of ionic crystals

Luigi Galgani
Department of Mathematics, Milan University, Milan, Italy

I report on a very recent result by F. Gangemi, A. Carati, L. Galgani, R. Gangemi, and A. Maiocchi. We consider a three dimensional model of a ionic crystal (Lithium Fluoride) and determine the motions of the ions by standard Molecular Dynamics simulations for 4096 particles. Then, through Kubo's formula involving essentially the time autocorrelation function of electric polarization, we compute the refractive index. The theoretical results at 300 K (of a completely classical type) agree in a surprisingly good way with the experimental data.

Stokes phenomenon and singularly perturbed differential equations

Vassili Gelfreich
(in collaboration with L. Lerman, R. Barrio, A. Champneys, and T. Lázaro)
Mathematics Institute, University of Warwick, Coventry, UK

In this talk we discuss relations between geometric singular perturbation theory, bifurcation theory and Stokes phenomenon. We discuss some recent developments of the theory, an application to the Hamiltonian-Hopf bifurcation and illustrate general theory with examples.

The singular perturbation theory studies slow manifolds in singularly perturbed differential equations. In the Hamiltonian case the slow manifold can be normally elliptic and therefore is not stable under a small perturbation. The disappearance of the invariant manifolds is accompanied by the disappearance of homoclinic orbits, i.e. the separatrix of the slow subsystem splits. In the analytic theory, the separatrix splitting is exponentially small with respect to the perturbation parameter.

The study of the separatrix splitting naturally leads to the Stokes phenomena where the asymptotic behaviour of analytic functions differ in different regions of the complex plane.

It is well known that bifurcation problems can often be studied with the help of singular perturbation theory. In these problems invariant manifolds and solutions can be found in the form of formal series. The series typically diverge and relations with analytical solutions is described by the Stokes phenomenon.

Arnold's mechanism of diffusion in the spatial circular restricted three-body problem

Marian Gidea
(in collaboration with Amadeu Delshams and Pablo Roldán)
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We consider the spatial circular restricted three-body problem, on the motion of an infinitesimal body in the Sun-Earth system. This can be described by a three-degree of freedom Hamiltonian system. We fix an energy level close to that of the collinear libration point L_1 located between Sun and Earth. Near L_1 there exists a normally hyperbolic invariant manifold, diffeomorphic to a three-sphere. Trajectories lying on this three-sphere are characterized by an out-of-plane amplitude that can vary only slightly.

Nevertheless, we show that we can obtain trajectories whose out-of-plane amplitude changes significantly by alternatively following trajectories of the flow restricted to the three-sphere and homoclinic trajectories that turn around the Earth. We provide an abstract theorem for the existence of diffusing trajectories, and numerical evidence that the premises of the theorem are satisfied in the three-body problem considered here.

The geometric mechanism underlying this construction is reminiscent of the Arnold diffusion problem for Hamiltonian systems. Our arguments, however, do not involve transition chains of tori as in the classical example of Arnold. We exploit mostly the 'outer dynamics' along homoclinic trajectories, and use very little information on the 'inner dynamics' restricted to the three-sphere. We explore a few energy levels and estimate the largest orbital inclination that can be achieved with our construction.

Mixed dynamics as a typical form of dynamical chaos for reversible systems

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We say that a system possesses a mixed dynamics if 1) it has infinitely many hyperbolic periodic orbits of all possible types (stable, unstable, saddle) and 2) the closures of the sets of orbits of different types have nonempty intersections. Recall that Newhouse regions are open domains (in the space of smooth dynamical systems) in which systems with homoclinic tangencies are dense. Newhouse regions in which systems with mixed dynamics are generic (compose residual subsets) are called absolute Newhouse regions. Their existence was proved in the paper [1] for the case of 2d diffeomorphisms close to a diffeomorphism with a nontransversal heteroclinic cycle containing two fixed (periodic) points with the Jacobians less and greater than 1. Fundamentally that "mixed dynamics" is the universal property of reversible chaotic systems. Moreover, in this case generic systems from absolute Newhouse regions have infinitely many stable, unstable, saddle and elliptic periodic orbits [2, 3].

As well-known, reversible systems are often met in applications and they can demonstrate a chaotic orbit behavior. However, the phenomenon of mixed dynamics means that this chaos can not be associated with "strange attractor" or "conservative chaos". Attractors and repellers have here a nonempty intersection containing symmetric orbits (elliptic and saddle ones) but do not coincide, since periodic sinks (sources) do not belong to the repeller (attractor). Therefore, "mixed dynamics" should be considered as a new form of dynamical chaos posed between "strange attractor" and "conservative chaos".

These and related questions will be discussed in the talk.

[1] S. V. Gonchenko, D. V. Turaev, and L. P. Shilnikov, On Newhouse domains of two-dimensional diffeomorphisms with a structurally unstable heteroclinic cycle, *Proc. Steklov Inst. Math.* **216** (1997) 70–118.

[2] J. S. W. Lamb and O. V. Stenkin, Newhouse regions for reversible systems with infinitely many stable, unstable and elliptic periodic orbits, *Nonlinearity* **17** (2004) 1217–1244.

[3] A. Delshams, S. V. Gonchenko, V. S. Gonchenko, J. T. Lázaro, and O. Sten'kin, Abundance of attracting, repelling and elliptic periodic orbits in two-dimensional reversible maps, *Nonlinearity* **26** (2013) 1–33.

Coherent Lagrangian vortices: The KAM tori of turbulence

George Haller

Institute of Mechanical Systems, ETH Zürich, Switzerland

Lagrangian Coherent Structures (LCSs) are special invariant manifolds that act as organizing centers in the extended phase space of a non-autonomous dynamical system. In this talk, I discuss elliptic LCSs that generalize the concept of Kolmogorov-Arnold-Moser (KAM) tori to temporally aperiodic dynamical systems over finite time scales. I show how elliptic LCSs delineate coherent material vortices in geophysical data sets, including satellite altimetry of the ocean and cloud-video footage of Jupiter.

On the dynamics of a solar sail

Àngel Jorba

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Dynamical systems have proven to be a useful tool for the design of space missions. For instance, the use of invariant manifolds is now common to design transfer strategies. Solar Sailing is a new form of spacecraft propulsion, where large membrane mirrors take advantage of the solar radiation pressure to push the spacecraft. Although the acceleration produced by the radiation pressure is smaller than the one achieved by a traditional spacecraft it is continuous and unlimited. This makes some long term missions more accessible, and opens a wide new range of possible applications that cannot be achieved by a traditional spacecraft.

In this presentation we will focus on the dynamics of a Solar sail in a couple of situations. We will introduce this problem focusing on a Solar sail in the Earth-Sun system. In this case, the model used will be the Restricted Three Body Problem (RTBP) plus Solar radiation pressure. The effect of the solar radiation pressure on the RTBP produces a 2D family of "artificial" equilibria, that can be parametrised by the orientation of the sail. We will describe the dynamics around some of these “artificial” equilibrium points. We note that, due to the solar radiation pressure, the system is Hamiltonian only for two cases: when the sail is perpendicular to the Sun - Sail line; and when the sail is aligned with the Sun - sail line (i.e., no sail effect). The main tool used to understand the dynamics will be the computation of centre manifolds, both for Hamiltonian and non-Hamiltonian cases.

The second example is the dynamics of a Solar sail close to an asteroid. Note that, in this case, the effect of the sail becomes very relevant due to the low mass of the asteroid. We will use, as a model, a Hill problem plus the effect of the Solar radiation pressure, and we will describe some aspects of the natural dynamics of the sail.

Stochastic Arnold diffusion of deterministic systems

Vadim Kaloshin

(in collaboration with O. Castejón, M. Guardia, J. Zhang, and K. Zhang)

University of Maryland, College Park, USA

In 1964 V. Arnold constructed an example of nearly integrable system exhibiting instabilities. In the 1970s physicist B. Chirikov coined the term for this phenomenon “Arnold diffusion”, where diffusion refers to stochastic nature of instability.

One of most famous example of stochastic instabilities for nearly integrable systems is dynamics of Asteroids in Kirkwood gaps in the Asteroid belt. They were discovered numerically by astronomer J. Wisdom. During the talk we describe a class of nearly integrable deterministic systems with stochastic diffusive behavior. More exactly, we show that distributions given by deterministic evolution of certain random initial conditions weakly converge to a diffusion process.

Slow-fast dynamics of a Duffing type equation: a case of study

Lev M. Lerman
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Slow varying differential system of the Duffing type

$$\dot{x} = y,\ \dot{y} = -\sin\theta - x\cos\theta - x^3,\ \dot{\theta} = \varepsilon \tag{1}$$

with Hamiltonian

$$H = \frac{y^2}{2} + \frac{x^4}{4} + \frac{x^2}{2}\cos\theta + x\sin\theta$$

demonstrates all possible types of behavior possible for a one and a half degree of freedom Hamiltonian system. The goal of the talk is to explain this behavior using the tools available by now in the theory of two dimensional symplectic diffeomorphisms but not only those. Additional work is to be done to explain sharp transitions from a more or less regular behavior to a chaotic mess. To this end, we draw the blow-up technique. Of course, it is not possible to present rigorous explanations of the chaotic behavior observed in the system. No tools exist nowadays that allows to give more or less satisfactory picture of the motion in the chaotic zones for a smooth symplectic 2-dimensional diffeomorphism.

We deliberately examine a slow fast Hamiltonian system that is rather simple in its form and has a minimally possible number of degenerate equilibria of simplest type (parabolic ones) for a frozen system. The system under consideration is in addition reversible w.r.t. involution L of the phase space acting as follows $L : (x, y, \theta) \to (-x, y, -\theta)$.

The frozen system is obtained when one considers θ to be a parameter of the system. The system with small $\varepsilon > 0$ is that with a slow varying parameter, it has a global cross-section $\theta = 0$, therefore one can study the related Poincaré map.

To investigate the dynamics we use various tools: the results on the almost integrable normal for the Hamiltonian near its almost elliptic slow manifold, the Fenichel results on the existence of hyperbolic slow manifold, blow up technique to represent the orbit transition near the disruption points, for the case of Hamiltonian system this is intimately connected with different solutions of the first Painleve equation.

This study allowed us to find for the related Poincaré map:

1. The region where there is an eternal adiabatic invariant;
2. A disk-shaped region where the dynamics is chaotic, Lyapunov exponent numerically found is positive, this region has infinitely many hyperbolic periodic orbits;
3. Existence of relaxation symmetric periodic orbits which some finite time pass near unstable hyperbolic part of the slow curve, like for canard periodic orbits;
4. Infinitely many bifurcations of symmetric periodic orbits of different types.

Equivariant symmetries in Poncelet maps

Hector E. Lomelí

Department of Mathematics, University of Texas, Austin TX, USA

We study symmetries of families of circle maps that arise in connection with the classic theorem of Poncelet.

We use the recent technique of *reduction by lifting* to study diffeomorphisms that have one-parameter families of continuous equivariant symmetries. Reduction by lifting can be used when a symmetry of a map is a flow with a global Poincaré cross section. In that situation, a topological covering space is introduced. As a consequence, we prove the existence of coordinates in which the map takes a reduced, skew-product form and hence allows for reduction of dimensionality.

We show that Poncelet maps have equivariant symmetries in which reduction by lifting can be used. The corresponding skew-product form is found. In particular, a formula for the rotation number is found in terms of elliptic functions.

From the Birkhoff theory of twist maps to Arnold diffusion in the a priori unstable case

Jean-Pierre Marco

IMJ-PRG, Université Paris 6, Paris, France

In this talk we exhibit a large class of finitely differentiable three-degree-of-freedom *a priori* unstable systems for which one can prove the generic existence of Arnold diffusion by a purely geometric method, based on a joint work with Marian Gidea. This method generalizes that introduced by Moeckel to prove the generic drift for families of twist maps on the annulus.

The Hamiltonians we consider possess a normally hyperbolic cylinder, diffeomorphic to $\mathbb{T}^2 \times \mathbb{R}$ contained in the 0 energy level, which itself admits a global section diffeomorphic to the annulus $\mathbb{T} \times \mathbb{R}$. The homoclinic connections yield a correspondence from the section to itself. We prove that generically the bisystem formed by the return map on the section together with the homoclinic correspondence admit drifting pseudo-orbits, which can be shadowed to get genuine orbits of the Hamiltonian flow. The talk is based on a joint work with Laurent Lazzarin.

Perturbing the cat map: mixed elliptic and hyperbolic dynamics

James Meiss

(in collaboration with Lev Lerman)

Departament of Mathematics, Boulder University, Boulder (CO), USA

Arnolds cat map is a prototypical dynamical system on the torus with uniformly hyperbolic dynamics. Since the famous picture of a scrambled cat in the 1968 book by Arnold and Avez, it has become one of the icons of chaos. In 2010, Lerman studied a family of maps homotopic to the cat map that has, in addition to a saddle, a parabolic fixed point. Lerman conjectured that this map could be a prototype for dynamics with a mixed phase space, having positive measure sets of nonuniformly hyperbolic and of elliptic orbits. We present some numerical evidence that supports Lermans conjecture. The elliptic orbits appear to be

confined to a pair of channels bounded by invariant manifolds of the two fixed points. The complement of the channels appears to be a positive measure Cantor set. Computations show that orbits in the complement have positive Lyapunov exponents.

On mechanisms of destruction of adiabatic invariance

Anatoly Neishtadt

Loughborough University, Loughborough, United Kingdom

In many problems of classical mechanics and theoretical physics dynamics can be described as a slow evolution of periodic or quasi-periodic processes. Adiabatic invariants are approximate first integrals for such a dynamics. Existence of adiabatic invariants makes dynamics close to regular. Destruction of adiabatic invariance leads to chaotic dynamics. In the talk it is planned to present a review of some mechanisms of destruction of adiabatic invariance. It is planned to consider examples of manifestation of these mechanisms in problems related to charged particles dynamics.

Dynamics of a ping-pong model

Rafael Ortega

(in collaboration with Markus Kunze)

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The Fermi-Ulam ping-pong model describes the motion of a particle (ball) colliding with two parallel walls (rackets). It is assumed that there is elastic bouncing and the motion of the rackets $x = w_i(t)$, $i = 1, 2$, is known. We are interested in the so-called Fermi acceleration which occurs when the velocity of the ball grows towards infinity. The periodic case, $w_i(t + T) = x_i(t)$, has been studied by several authors. This talk will be about the quasi-periodic case, specially when no diophantine condition is satisfied.

Stability interchanges in a curved Sitnikov problem

Ernesto Pérez-Chavela

(in collaboration with Luis Franco-Pérez, Marian Gidea, and Mark Levi)

Department of Mathematics, UAM, Iztapalapa, México

We consider a curved Sitnikov problem, in which an infinitesimal particle moves on a circle under the gravitational influence of two equal masses in Keplerian motion within a plane perpendicular to that circle. There are two equilibrium points, whose stability we are studying. We show that one of the equilibrium points undergoes stability interchanges as the semi-major axis of the Keplerian ellipses approaches the diameter of that circle. To derive this result, we first formulate and prove a general theorem on stability interchanges, and then we apply it to our model.

Chaotic fluid mixing

Vered Rom-Kedar

Department of Computer Science and Applied Mathematics, Weizmann Institute of Science, Rehovot, Israel

Some mathematical principles that arise when one utilizes dynamical system tools to study mixing of passive scalars in unsteady fluid flows will be surveyed. It turns out that the beloved Melnikov function methodology provides many insights into this subject, beyond the perturbative regime for which it was originally designed. In particular, the role of the stable and unstable manifolds in governing transport [5] will be explained (with examples of recent extensions of these concepts), the universality in the scaling of the homoclinic tangles [3, 4] will be explored, the role of weak three dimensionality on transport of surface particles will be demonstrated [2], and a new indicator for studying mixing properties of systems with mixed phase-space will be presented [1].

[1] R. Mundel, E. Fredj, H. Gildor, and V. Rom-Kedar, New Lagrangian diagnostics for characterizing fluid flow mixing, *Phys. of Fluids* **26** (2014) 126602.

[2] R. Aharon, V. Rom-Kedar, and H. Gildor, When complexity leads to simplicity: Ocean surface mixing simplified by vertical convection, *Phys. Fluids* **24** (2012) 056603.

[3] V. Rom-Kedar, Frequency spanning homoclinic families, *Commun. Nonlinear Sci. Numerical Simul.* **8** (2003) 149–169.

[4] V. Rom-Kedar and A. C. Poje, Universal properties of chaotic transport in the presence of diffusion, *Phys. Fluids* **11** (1999) 2044–2057.

[5] V. Rom-Kedar, A. Leonard, and S. Wiggins, An analytical study of transport, mixing and chaos in an unsteady vortical flow, *J. Fluid Mech.* **214** (1990), 347–394.

Gevrey examples in Hamiltonian perturbation theory

David Sauzin

(in collaboration with L. Lazzarini and J.-P. Marco.)

CNRS UMI 3483 - Fibonacci Laboratory, Pisa, Italy

We consider exact symplectic perturbations of integrable diffeormorphisms on the cotangent bundle of the torus, in the analytic or Gevrey category. An integrable diffeormophism cannot have nonempty wandering domains. For near-integrable systems, we estimate the size of wandering domains (Lebesgue measure or Gromov capacity): upper estimates are connected with Nekhoroshev theory (they are exponentially small), lower estimates are connected with examples of Arnold diffusion. The construction of the latter examples is done in the Gevrey non-analytic case, based on a “coupling lemma” orginally due to M.Herman.

Relative equilibria of the Restricted Three Body Problem in curved spaces

Carles Simó

Department de Matemàtica Aplicada i Anàlisi, Universitat de Barcelona, Barcelona, Spain

We use a formulation of the n-body problem in spaces of constant Gaussian curvature, $\kappa \in \mathbb{R}$ as widely used by F. Diacu and coworkers. We consider $\mathbb{S}^2$ with arbitrary $\kappa > 0$ (resp. $\mathbb{H}^2$ with arbitrary $\kappa < 0$) in a formulation which is also valid for the planar case $\kappa = 0$.

Several general results and limit cases as well as the stability properties are considered in general including the analysis of some limit cases. The relevant parameters are κ and the mass ratio μ. The study is completed numerically using continuation from the planar problem and from other limit cases. For concreteness in the case $\kappa > 0$ we restrict the study to the three bodies at the upper hemisphere.

Some surprising phenomena, like the coexistence of several triangular-like solutions for some values (κ, μ) and many stability changes will be discussed.

KAM in infinite dimensions

Yannick Sire

(in collaboration with Rafael de la Llave and Ernest Fontich)

Centre de Mathématique et Informatique, Université Aix-Marseille, Marseille, France

I will describe several works about KAM theorems in infinite dimensions. A first one deals with the case of infinite lattices and a second one with some PDEs coming fluid mechanics. The methods are new and allow to construct several types of invariant tori (including infinite dimensional tori on lattices).

On the Homogenization of Hamilton-Jacobi equation

Alfonso Sorrentino

Department of Mathematics, Università degli Studi di Roma "Tor Vergata", Rome, Italy

In this talk I would like to discuss the problem of homogenizing Hamilton-Jacobi equation, in the case of Tonelli Hamiltonians which are invariant under the action of a discrete group. This different point of view, besides providing a general setting that embraces previous results in the literature, will allow us to get a clearer understanding of the geometry of the limit space and of the structure of the homogenized problem.

Locally linear billiard maps

Dmitry V. Treschev

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Can a billiard map be locally linear near a periodic orbit of period 2? This question can be asked in all dimensions beginning from 2. It is more interesting to consider the situation when the linear symplectic map to which the billiard map is locally conjugated, is totally elliptic. Then the billiard is locally integrable.

Answer to this question is still unknown. We reduce the question to analysis of some equations which have formal solutions, but conjugacy of the corresponding series is still unproven. Numerical results suggest a conjecture that such a conjugacy takes place and several other conjectures which will be discussed in the talk.

Fermi acceleration in non-ergodic systems

Dmitry Turaev

(in collaboration with V. Gelfreich, T. Pereira, V. Rom-Kedar, K. Shah, and A. Vladimirov)

Department of Mathematics, Imperial College, London, UK

We consider a homogeneous Hamiltonian system subject to a slow periodic variation of parameters. We show that the non-ergodicity with respect to the Liouville measure leads to a steady increase of the entropy of the system, which leads to an exponential growth of energy. We discuss this phenomenon for the examples of billiards with periodically moving boundaries, billiards with holes, relativistic billiards, and a field in a resonator of time-varying shape.

Some models arising in crystal dislocations

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We consider an evolution equation arising in the Peierls-Nabarro model for crystal dislocation. We study the evolution of such dislocation function and show that, at a macroscopic scale, the dislocations have the tendency to concentrate at single points of the crystal, where the size of the slip coincides with the natural periodicity of the medium.

These dislocation points evolve according to the external stress and an interior potential which can be either repulsive or attractive, according to the orientation or the dislocations.

In case of attractive potentials, collision may occurs, but the system can be described after the collision time and it possesses exponential relaxation times.

Though the problem seems of local nature, the leading order of the diffusion turns out to be a nonlocal integrodifferential operator.

Melnikov-type method for splitting of separatrices for an explicit range of small parameter

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One of the main tools to determine the existence of (or non-existence of) chaos in a perturbed hamiltonian system is the Melnikov theory. In this theory, the distance between the stable and unstable manifolds of the perturbed system is calculated up to the first order term, hence the precise range of small parameter for which the transversal intersection exists is unknown.

We propose the method which allows to compute an explicit range of small parameter for which the intersection exists, with the hope that to obtain the size of the parameter from which the continuation with other direct geometric tools is possible. The method is computer assisted (a computer assisted proof). We applied it to the system

$$(x', y') = \left(y - \varepsilon \cos(t) y^2, x - x^2\right),$$

given by the Hamiltonian $H_\varepsilon(x, y) = (y^2 - x^2)/2 + x^3/3 - \varepsilon y^3 \cos t\ /3$.

At the present stage of the project we have the proof of the existence of topologically transversal intersection for $\epsilon \in (0, 10^{-4}]$. The work toward the standard transversality is in progress.

To compute the Melnikov distance our method combines two ingredients, both computer assisted

- geometric method to establish explicit bounds for normally hyperbolic invariant manifolds (NHIM) and their stable and unstable fibers, together with their dependence of parameter. The NIHM in question in our example the periodic orbits $(0, 0, t)$.
- the rigorous C^2-integration of our system away from the NHIM.

The method can be generalized to many dimensions.

The dynamics of dominant Hamiltonian systems

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Near a maximal resonance of a nearly integrable Hamiltonian system, the local dynamics is approximated by a mechanical system called the "slow system". The generic property of such systems is important for understanding Arnold diffusion in higher dimensions. We call a slow system "dominant" if the associated resonance group admits a subgroup of low order resonances (called "strong"), and the the rest has much higher order (called "weak"). We show a dominant system is well approximated by a lower dimensional mechanical system obtained from the strong resonances, both in the sense of vector field and weak KAM solutions. In particular, this theory allows us to construct an admissible path for Arnold diffusion in arbitrary degree of freedom.

Communications

On limit cycles in transformed Hamiltonian system

Svetlana Atslega

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We study perturbed Hamiltonian systems of the form

$$\begin{cases} x' = \dfrac{\partial H}{\partial y} + a(x,y)\dfrac{\partial H}{\partial x}, \\ y' = -\dfrac{\partial H}{\partial x} + b(x,y)\dfrac{\partial H}{\partial y}. \end{cases}$$

and show (on examples) how multiple limit cycles of various shapes appear in these systems and how this process can be controlled by the choice of functions $a(x,y)$ and $b(x,y)$.

[1] S. Atslega and F. Sadyrbaev, On periodic solutions of Liénard Type Equations, *Mathematical Modelling and Analysis* **18** (2013) 708–716.

[2] T. Johnson and W. Tucker, An improved lower bound on the number of limit cycles bifurcating from a Hamiltonian planar vector field of degree 7, *Int. Journal of Bifurcation and Chaos* **20** (2010) 1451–1458.

[3] R. Winkel, A trasfer principle in the real plane from nonsingular algebraic curves to polynomial vector fields, *Geometriae Dedicata* **79** (2000) 101–108.

Global organization of phase space in 4D symplectic maps

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For a generic 4D symplectic map we visualize the global organization of regular tori using 3D phase-space slices [1, 2]. The regular 2-tori are shown to be arranged around a skeleton of elliptic 1-tori. The 1-tori occur in two types of one-parameter families: Lyapunov families emanating from elliptic-elliptic periodic orbits and families originating from rank-1 resonances. At resonance gaps of both types of families periodic orbits may occur, similar to the Poincare-Birkhoff theorem for 2D maps. Based on this we describe the hierarchical structure of regular tori in the 4D phase space. Each level of the hierarchy can result from three types of structures which generalizes the islands-around-islands hierarchy in 2D maps. Furthermore, we demonstrate that bifurcations of families 1-tori can be understood [3] using normal forms for normal-internal resonances in quasi-periodically driven systems [4].

[1] M. Richter, S. Lange, A. Bäcker, and R. Ketzmerick, Visualization and comparison of classical structures and quantum states of 4D maps, *Phys. Rev. E* **89** (2014) 022902.

[2] S. Lange, M. Richter, F. Onken, A. Bäcker, and R. Ketzmerick, Global structure of regular tori in a generic 4D symplectic map, *Chaos* **24** (2014) 024409.

[3] F. Onken, S. Lange, R. Ketzmerick, and A. Bäcker, Bifurcations of 1D tori in 4D symplectic maps (in preparation).

[4] H. Broer, H. Hanßmann, À. Jorba, J. Villanueva, and F. Wagener, Normal-internal resonances in quasi-periodically forced oscillators: a conservative approach, *Nonlinearity* **16** (2003) 1751–1791.

Exponentially small splitting of separatrices for 3D whiskered tori with cubic frequencies

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We study the splitting of invariant manifolds of 3-dimensional whiskered (hyperbolic) tori with three frequencies in nearly-integrable Hamiltonian systems.

We consider tori with a frequency vector of the form $\omega = (1, \Omega, \Omega^2)$, where Ω is a cubic irrational number. We pay special attention to the case in which Ω is the cubic golden number, the real root of $x^3 + x - 1 = 0$. We show that the Poincaré-Melnikov method can be applied to establish the existence of homoclinic orbits to the whiskered tori and prove that these homoclinic orbits are transverse. Thereby, we generalize the results obtained in previous papers for 2-dimensional whiskered tori with a quadratic frequency vector.

On phenomenon of mixed dynamics in the Pikovsky-Topaj models of coupled rotators

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In 2002 year A. Pikovsky and D. Topaj observed an interesting dynamical property in the system describing the dynamics of symmetrically coupled rotators [1]. For the small values of a coupled parameter the dynamics in the system look like a conservative one. In this case an attractor and a repeller (constructed from the symmetry line) coincide. However with increasing of the coupled parameter the dynamics in the system cease to be conservative and an asymmetry between an attractor and a repeller appear.

In the present talk we describe the bifurcations of symmetry breaking in the Pikovsky-Topaj system. We show that these bifurcations leads to the appearing of a such interesting kind of chaos as *mixed dynamics*. Moreover we explain the reason for the numerically observed asymmetry of the attractor and repeller which appears with the increasing of the coupling parameter.

[1] D. Topaj and A. Pikovsky, Reversibility vs. synchronization in oscillator lattices, *Physica D* **170** (2002) 118–130.

Nonpersistence of resonant curves in perturbed geodesic circular billiards

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Let $r = r(\theta)$ be a strictly convex curve given in polar coordinates and $T(\theta, \varphi)$ be its associated billiard map. A (k, n)-resonant curve γ is a graph $\varphi = g(\theta)$, invariant by T and such that every orbit on γ is n-periodic with rotation number k/n.

In [1], R. Ramírez-Ros proved that if a perturbation of the circle is given by $r_\epsilon(\theta) = 1 + \epsilon r_1(\theta) + O(\epsilon^2)$, with $r_1(\theta) = \sum_{j\in\mathbb{Z}} c_j e^{ij\theta}$ is such that, for $n \geq 2$ there exists $j \in n\mathbb{Z} \setminus \{0\}$ with $c_j \neq 0$ then there will be no (k, n)-resonant curve for its associated billiard map, for every k, $0 < k/n \leq 1/2$, k and n relatively primes.

In this joint work with L. Coutinho dos Santos, we show that the same result is true for perturbations of the geodesic circular billiard on surfaces of constant curvature.

[1] R. Ramírez-Ros, Break-up of resonant invariant curves in billiards and dual billiards associated to perturbed circular tables, *Physica D* **214** (2016) 78–87.

[2] L. Coutinho dos Santos and S. Pinto-de-Carvalho, Periodic orbits of oval billiards on surfaces of constant curvature, arXiv:1411.0236v2 (2014).

Stability and bifurcation of coherent structures in nonlinear lattices

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In this talk, we present our recent result ralted with stability and bifurcation of discrete breathers in nonlinear lattices. We study the existence and stability of multibreathers in Klein-Gordon chains with long range interactions. We provide a general framework where such long range effects can be taken into consideration for arbitrarily varying (as a function of the node distance) linear couplings between arbitrary sets of neighbors in the chain. Next, we obtain a general criterion for spectral stability of multi-site breathers for a small coupling constant for a nonlinear metamaterial lattice. We show how this criterion differs from the one derived in the Klein-Gordon lattice. The existence and uniqueness results of periodic and asymptotic travelling waves of the nonlinear metamaterial lattice are presented. Employing Melnikov theory for an advance-delay functional differential equation. The existence and the stability of periodic and asymptotic waves are also computed and discussed numerically.

Cascades of bifurcations in quasiperiodic perturbations of 1d maps

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In this work we deal with the effect of a quasi-periodic perturbation on a cascade of period doubling bifurcations of one-dimensional maps. We notice that after the quasi-periodic perturbation no periodic points can exist and they are replaced (when they exist) by closed invariant curves. Moreover, an invariant curve is reducible if its linearized normal behaviour is reducible.

Let g_α be a one-parameter family of one-dimensional maps with a cascade of period doubling bifurcations. If ϵ is a perturbing parameter of this family, we can prove under mild conditions that from each value α of the parameter for which g_α has a superattracting periodic point, two reducibility loss bifurcation curves (for which the invariant curve changes from reducible to non-reducible) are born in the parameter plane (α, ϵ). This means that these curves are present for all the cascade [1].

[1] À. Jorba, P. Rabassa, and J. C. Tatjer, Superstable periodic orbits of 1d maps under quasi-periodic forcing and reducibility loss, *Discrete Continuous Dynamical Syst. Ser. B* **34** (2014), 589–597.

The higher-dimensional Poincar-Birkhoff theorem for Hamiltonial systems

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We propose a higher dimensional generalization of the Poincar-Birkhoff Theorem which applies to Poincar time maps of Hamiltonian systems. The maps under consideration are neither required to be close to the identity nor to have a monotone twist. The annulus is replaced by the product of an N-dimensional torus and the interior of an embedded sphere in the N-dimensional Euclidean space; on the other hand, the classical boundary twist condition is replaced by an avoiding rays condition.

Posters

Characterizing island structures in area-preserving maps

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A newly suggested diagnostic family [1] is used to study the structure of the phase space of area-preserving chaotic maps. The phase space typically consists of islands bounded by segments of the stable and unstable manifolds of two hyperbolic periodic points, and a chaotic sea. Islands can have a rich variety of structures, such as fixed points, periodic orbits, chaotic regions, and smaller islands. The diagnostic family encompasses a number of asymptotic extremal values denoted by M_ϕ^+, M_ϕ^-, M_ϕ^{shift}, and M_ϕ^{mean}, where ϕ is an observable. Using those extrema fields we deduce information about the dynamics of area-preserving chaotic maps, and we are able to detect smaller islands up to a certain size which depends on the resolution used. Furthermore, we use the cumulative distribution function (CDF) and the probability distribution function (PDF) of the extremal values to distinguish different islands along with some of their properties such as their number, sizes, and positions. We also examine the convergence in time of the extrema fields and their CDFs. We apply these methods in doubly-periodic domains ($\mathbb{T}^2$) as well as in open domains ($\mathbb{R}^2$). We test our results on several maps, such as the Chirikov standard map, the sawtooth map, Hénon map, and other cubic maps.

[1] R. Mundel, E. Fredj, H. Gildor, and V. Rom-Kedar, New Lagrangian diagnostics for characterizing fluid flow mixing, *Physics of Fluids* **26** (2014) 126602.

Homoclinic chaos in a pair of parametrically-driven coupled SQUIDs

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An rf superconducting quantum interference device (SQUID) consists of a superconducting ring interrupted by a Josephson junction (JJ). When driven by an alternating magnetic field, the induced supercurrents around the ring are determined by the JJ through the celebrated Josephson relations. This system exhibits rich nonlinear behavior, including chaotic effects. We study the dynamics of a pair of parametrically-driven coupled SQUIDs arranged in series. We take advantage of the weak damping that characterizes these systems to perform a multiple-scales analysis and obtain amplitude equations, describing the slow dynamics of the system. This picture allows us to expose the existence of homoclinic orbits in the dynamics of the integrable part of the slow equations of motion. Using high-dimensional Melnikov theory, we are able to obtain explicit parameter values for which these orbits persist in the full system, consisting of both Hamiltonian and non-Hamiltonian perturbations, to form so called Silnikov orbits, indicating a loss of integrability and the existence of chaos.

Towards chaotic dynamics of a nonholonomic model of Celtic stone

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We consider a nonholonomic model of Celtic stone (rattleback) movement along the plane. As well-known, the movement of Celtic stone on the plane is considered still as one of most complicated and very little studied type of rigid body movements. Moreover, it is one of view types of such movements in which chaotic dynamics is possible. The existence of strange attractors in the celtic stone dynamics was recently discovered by A. V. Borisov and I. S. Mamaev [1]. In the paper [2] these results were extended and main bifurcations leading to chaos appearance were studied. In particular, various types of chaotic dynamics were found in the model: a spiral strange attractor, torus-chaos attractors, nearly conservative chaos and even the so-called mixed dynamics [3]. The latter type of chaotic orbit behavior means that the corresponding nonwandering set contains infinitely many coexisting periodic orbits of all possible types: stable, completely unstable, saddle and, due to reversibility of the system, symmetric elliptic periodic orbits. Moreover, for certain types of Celtic stones (possessing certain geometrical and physical properties) strange Lorenz-like attractors were found in their nonholonomic models. In this talk we observe these results.

[1] A. V. Borisov and I. S. Mamaev, Strange attractors in dynamics of celtic stones, *Advances in Physic Sciences* **117** (2003) 407–418.

[2] A. S. Gonchenko, S. V. Gonchenko, and A. O. Kazakov, On new aspects of chaotic dynamics of “celtic stone”, *Rus. Nonlin. Dyn.* **8** (2012) 507–518.

[3] S. V. Gonchenko, D. V. Turaev, and L. P. Shilnikov, On Newhouse domains of two-dimensional diffeomorphisms with a structurally unstable heteroclinic cycle, *Proc. Steklov Inst. Math.* **216** (1997) 70–118.

A fractalization route for affine skew-products on the complex plane

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Recall that Floquet Theory states that a linear ordinary differential equation with periodic coefficients can be reduced, by means of a periodic change of variables, to constant coefficients. The question of when a linear system, induced by a map or by an ODE, is reducible or not becomes much more interesting in the quasi-periodic setting, where there are non-reducible systems. Besides providing criteria to contradistinguish reducible and non-reducible systems, it is also of interest to see how non-reducibility manifests in dynamics. In this work we study a special kind of maps, the so-called quasi-periodic skew-products on the complex plane. We show that, in this situation, the only source of non-reducibility comes from a topological obstruction. We also study affine systems, the simplest systems in which one observes invariant curves. We show that non-reducibility has a visible impact in the bifurcation at zero Lyapunov exponent, that is, when the curves passes from attracting to repelling. We prove for some cases, that, when its linear behaviour is not reducible, and as the Lyapunov exponent goes to zero, the invariant curve gets destroyed by a mechanism of fractalization.

What is the origin of power-law trapping in 4D maps?

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While power-law trapping in 2D maps can be explained by a hierarchy of partial transport barriers [Meiss, Ott, PRL 1985], its origin in higher dimensional maps is still an open question. We study 4D symplectic maps with a regular region embedded in a large chaotic sea, i.e., far away from the near-integrable regime. Chaotic orbits are trapped in the vicinity of the regular region and show a power-law decay of survival times. We search for the trapping mechanism by visualizing the trapped orbits in 3D phase-space slices [1] and analyzing their time-dependent frequencies. While this has not yet revealed the mechanism, we can make the following statements about the power-law trapping: It is clearly different from trapping in 2D maps, as the trapped orbits do not explore the hierarchy [2] of the 4D phase space. Moreover, it is not related to the Arnol'd web.

[1] M. Richter, S. Lange, A. Bäcker, and R. Ketzmerick, Visualization and comparison of classical structures and quantum states of four-dimensional maps, *Phys. Rev. E* **89** (2014) 022902.

[2] S. Lange, M. Richter, F. Onken, A. Bäcker, and R. Ketzmerick, Global structure of regular tori in a generic 4D symplectic map, *Chaos* **24** (2014) 024409.

Monodromy and principal bundles

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It is known that 2DOF integrable Hamiltonian systems with focus-focus singularities, such as the spherical pendulum, the Lagrange top and the others, have non-trivial monodromy. Moreover, it has been shown that several Hamiltonian systems with different types of singularities and also integrable non-Hamiltonian systems admit non-trivial monodromy. The common aspect of all these systems is that there is a (semi-local) $\mathbb{S}^1$-symmetry. The $\mathbb{S}^1$-symmetry gives rise to a particular circle bundle, non-trivial Chern class of which implies non-trivial monodromy. This observation can be profoundly generalized. We will illustrate our results with a few examples of 2DOF and 3DOF systems admitting $\mathbb{T}^k$-symmetry, $k = 1, 2$.

Exponentially small splitting of separatrices in area-preserving maps around 1:3 resonance

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This study is divided in 2 parts. The first part is focused on maps exactly at 1:3 resonance, where the exponential splitting is shown using resurgence. The second part is focused on the unfolded maps. There complex matching shows that the dominant term comes from the splitting at the resonance.

Skeleton of regular tori and bifurcations in 4D symplectic maps

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To visualize the dynamics of 4D symplectic maps we use the recently introduced 3D phase space slices [1]. By this it is possible to illustrate the organization of regular 2D tori around a skeleton formed by families of elliptic 1D tori, which are either Lyapunov families of elliptic-elliptic periodic orbits or due to rank-1 resonances. As generic example system we study the 4D coupled standard map [2] and are able to describe a hierarchical structure analogously to the known island-around-island hierarchy in 2D maps.

It turns out that the families of 1D tori show gaps, bends and also branches which can be explained in terms of bifurcations, however without any parameter being varied. These observations are consistent with normal form results obtained by Broer [3] and Todesco [4]. In addition the skeleton of regular 1-tori is supplemented by the skeleton of hyperbolic 1-tori which is of particular relevance to chaotic transport.

[1] M. Richter, S. Lange, A. Bäcker, and R. Ketzmerick, Visualization and comparison of classical structures and quantum states of 4D maps, *Phys. Rev. E* **89** (2014) 022902.

[2] C. Froeschlé, Numerical study of a four-dimensional mapping, *Astron. & Astrophys.* **16** (1972) 172–189.

[3] H. Broer, H. Hanßmann, À. Jorba, J. Villanueva, and F. Wagener, Normal-internal resonances in quasi-periodically forced oscillators: a conservative approach, *Nonlinearity* **16** (2003) 1751–1791.

[4] E. Todesco, Analysis of resonant structures of four-dimensional symplectic mappings, using normal forms, *Phys. Rev. E* **50** (1994) R4298–R4301.

Exponentially small phenomena for the length spectrum in some billiard tables

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Let $q > 2$ be a period. There are at least two $(1, q)$-periodic trajectories inside any smooth strictly convex billiard table. We quantify the chaotic dynamics of these billiard tables close to their boundaries by measuring the differences of the lengths of their $(1, q)$-periodic trajectories as q tends to infinity.

On the one hand, we establish that these differences are exponentially small in q for analytic strictly convex billiard tables. The result is obtained in two steps. First, we prove a Neishtadt-like theorem, in which the billiard twist map is written as an integrable twist map plus an exponentially small remainder on the distance to the boundary. Second, we apply the MacKay-Meiss-Percival action principle. This result is contained in [1].

On the other hand, based on numerical experiments, we conjecture that, if the billiard table is a generic axisymmetric analytic strictly convex curve, then the differences of lengths between the $(1, q)$-axisymmetric periodic trajectories behave asymptotically like an exponentially small factor $q^{-3} \exp(-rq)$ times either a constant or an oscillating function, and the exponent r is half of the radius of convergence of the Borel transform of the well-known asymptotic series for the lengths of the $(1, q)$-periodic trajectories. Our experiments are focused on some perturbed ellipses and circles, so we can compare the numerical results with some analytical predictions obtained by Melnikov methods. We also detect some non-generic behaviors due to the presence of extra symmetries. Our computations require a multiple-precision arithmetic and have been programmed in PARI/GP. This study is contained in [2].

[1] P. Martín, R. Ramírez-Ros, and A. Tamarit-Sariol, On the length and area spectrum of analytic convex domains, arXiv:1410.1623 (2014).

[2] P. Martín, R. Ramírez-Ros, and A. Tamarit-Sariol, Exponentially small asymptotic formulas for the length spectrum in some billiard tables, arXiv 1504.08243 (2015).

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