# Codes and Cryptography

Jorge L. Villar

MAMME, Fall 2015

## PART XII

# Outline

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with a **chaining mode** can encrypt arbitrarily long messages

# Pseudorandom Generators (I)

$g : \{0,1\}^\kappa \to \{0,1\}^\ell$

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## Definition (Computational Indistinguishability)

Let $V = \{V_\ell\}_{\ell \in \mathbb{Z}^+}$, $W = \{W_\ell\}_{\ell \in \mathbb{Z}^+}$ be two families of probability distributions on a finite set family $\mathcal{X} = \{\mathcal{X}_\ell\}_{\ell \in \mathbb{Z}^+}$. We say that $V$ and $W$ are computationally indistinguishable if for any PPTM $\mathcal{D}$,

$$\left| \Pr[\mathcal{D}(1^\ell, V_\ell) = 1] - \Pr[\mathcal{D}(1^\ell, W_\ell) = 1] \right| \in \mathbf{negl}(\ell)$$

# Pseudorandom Generators (II)

NOTATION: $V \approx_c W$ means $V, W$ are computationally indistinguishable.
We abuse the notation writing $V_\ell \approx_c W_\ell$.
$U_\ell$ denotes the uniform distribution in $\{0,1\}^\ell$.

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**Definition (Pseudorandom Generator (PRG))**

An efficiently computable map family $g = \{g_\ell : \{0,1\}^{\kappa(\ell)} \to \{0,1\}^\ell\}_{\ell \in \mathbb{Z}^+}$, where $\kappa(\ell) \leq \ell$, is a pseudorandom generator if

$$g(U_{\kappa(\ell)}) \approx_c U_\ell$$

# Examples

- **Blum-Blum-Shub generator** (1986). Very slow. Provably secure (under QR Assumption)
  $g(s) = (r_1, r_2, \ldots, r_\ell)$ where $r_i = LSB(s^{2^i} \mod n)$ for $n = pq$, $p = 2p' + 1$ and $q = 2q' + 1$, $p', q'$ random primes of binary length $\kappa/2 - 1$.

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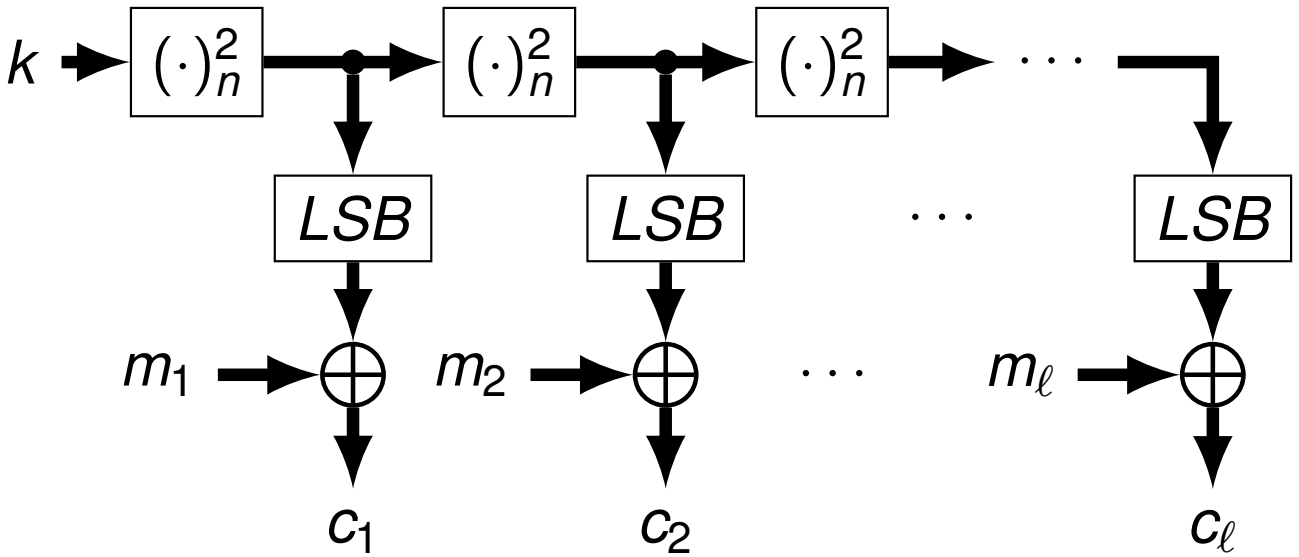
- **Non-Linear Feedback Shift Registers**. Very fast. Widely used (GSM, Bluetooth, ...). In general, they lead to weak symmetric encryption schemes.

# SE From Blum-Blum-Shub PRG



# Pseudorandom Permutations

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**Definition (Pseudorandom Permutation (PRP))**

A family of sets of efficiently computable bijective maps $\mathcal{F} = \{\mathcal{F}_\ell\}_{\ell \in \mathbb{Z}^+}$, $\mathcal{F}_\ell = \{f_k : \{0,1\}^\ell \to \{0,1\}^\ell\}_{k \in \mathcal{K}_\ell}$ is a family of pseudorandom permutations if for all oracle PPTM $\mathcal{D}$,

$$\mathcal{D}^{\text{Eval}}(1^\ell) \approx_c \mathcal{D}^{\text{Rand}}(1^\ell)$$

where $\text{Eval}(x) = f_k(x)$ for a randomly chosen $f_k \in \mathcal{F}_\ell$ and $\text{Rand}(x) = h(x)$ for $h$ randomly chosen among all permutations of $\{0,1\}^\ell$. ($f_k$ and $h$ are the same in all oracle queries.)

# Building Block Ciphers from PRP (I)

Let $\mathcal{F}$ be a pseudorandom permutation family with efficiently computable inverses, and $\Pi_{\mathcal{F}} = (\text{KeyGen}, \text{Enc}, \text{Dec})$ a symmetric encryption scheme such that

KeyGen($\ell$) :
**output** $k \leftarrow \mathcal{K}_\ell$;

Enc($k, m$) :
**output** $f_k(m)$;

Dec($k, c$) :
**output** $f_k^{-1}(c)$;

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▸ proof. . .

**Theorem**

*$\Pi_{\mathcal{F}}$ is SE-LR$_1$ secure but not SE-LR$_2$ secure (i.e., it is secure if and only if $q_{LR} \leq 1$).*

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$r \leftarrow \{0,1\}^\kappa$; //or instead, use a counter (stateful encryption)
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▸ proof...

**Theorem**

*If* $2^{-\kappa} \in \mathbf{negl}(\ell)$ *then* $\Pi_{\mathcal{F}}$ *is SE-LR secure*

# Practical Construction of Block Ciphers

Heuristic constructions based on:

- **confusion:** the plaintext/ciphertext pair depend on the key in an involved way
- **diffusion:** the ciphertext depends on the plaintext in an involved way
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GOAL: every input bit and every key bit affects all bits in the output

Structure:

- **key expansion:** a set of round keys is generated from the original key
- **mixing** of permutation (XORing, shifting...) and substitution (S-boxes) in each round.

# Example: AES256

Key size: 256 bits

Message and ciphertext block size: 128 bits ($4 \times 4$ byte matrix)

Number of encryption rounds: 14

Key expansion: from 256 bits to 1920 bits (15 subkeys, each one a $4 \times 4$ matrix of bytes) Adds 4 bytes in each iteration step

Encryption round:

- Apply a substitution box to each byte
- Perform rotation operations to rows
- Perform linear transformation to columns
- XOR with the round key

More details in
http://en.wikipedia.org/wiki/Advanced_Encryption_Standard

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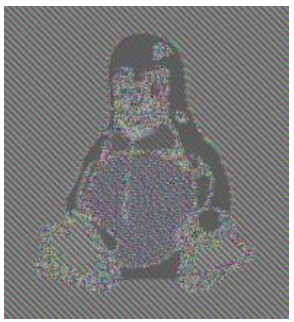

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Equal message blocks result in equal ciphertext blocks!

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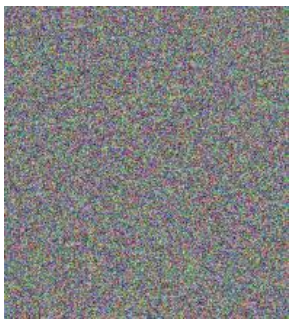
CBC operation mode:

$c_0 = iv$

$c_i = \mathrm{Enc}(k, m_i \oplus c_{i-1})$



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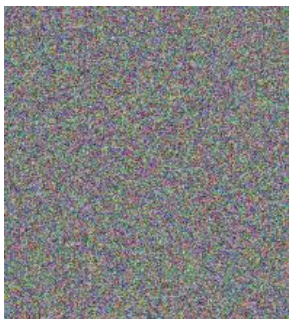

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The ciphertext has an extra random block $c_0$, the "initialization vector"

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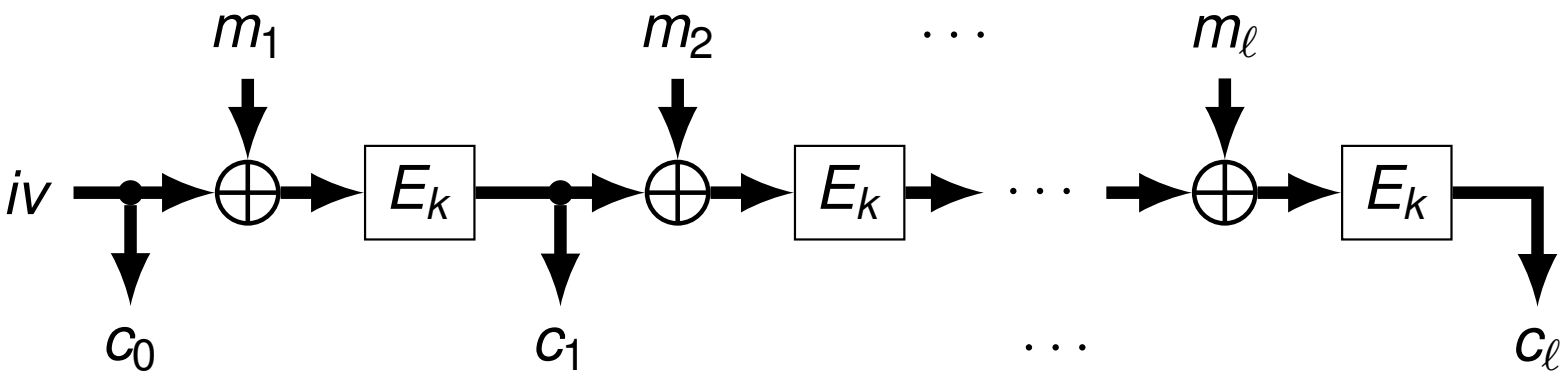

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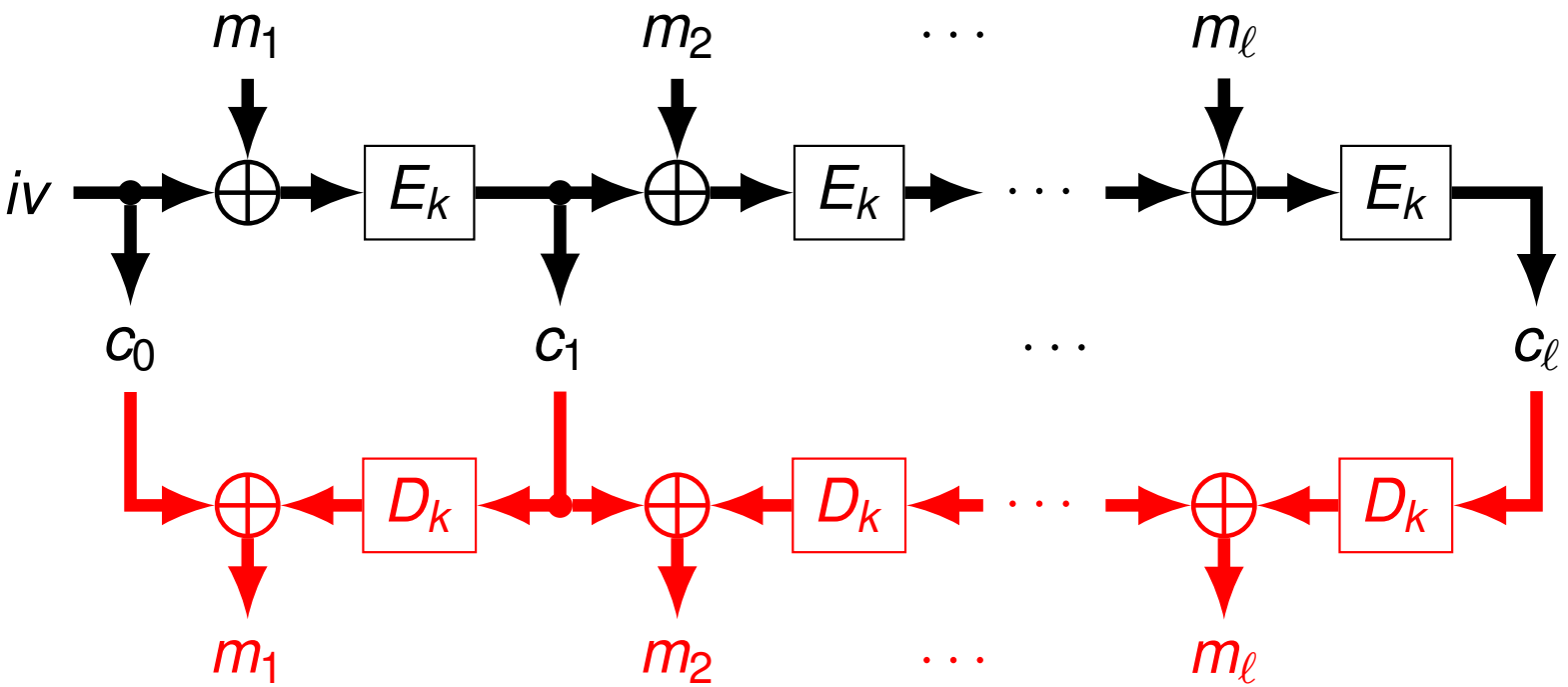
$m_1$
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$\cdots$
$m_\ell$
$iv$
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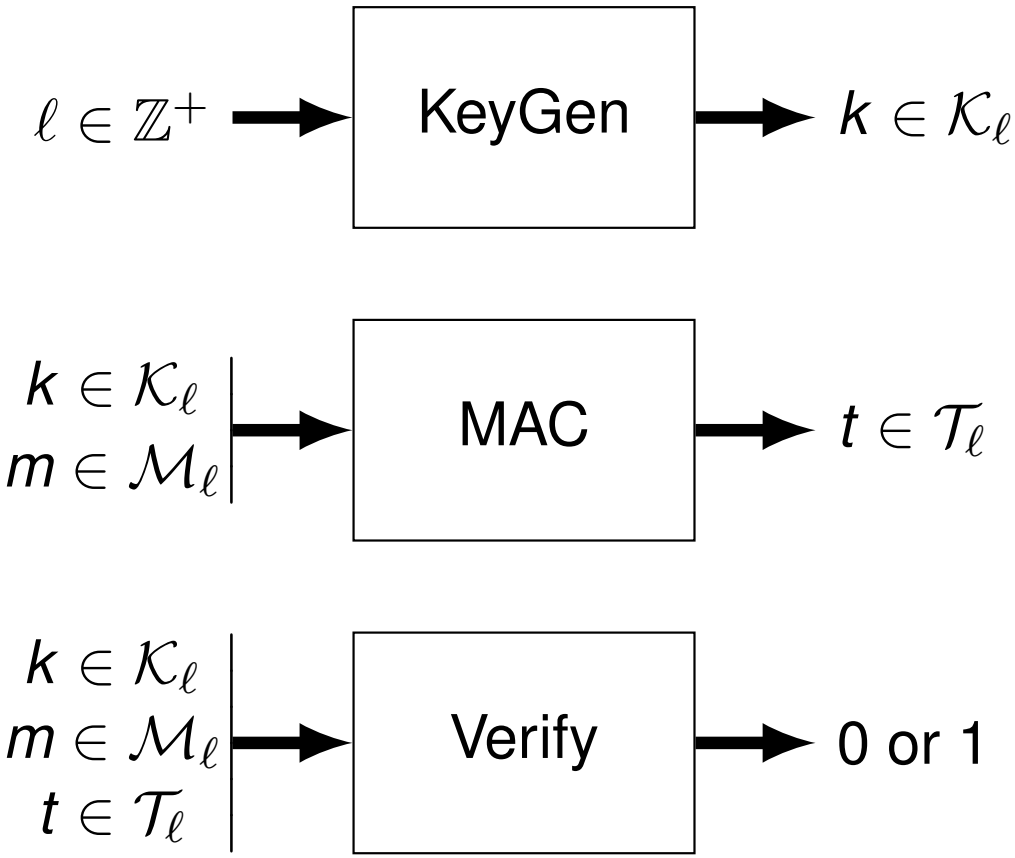
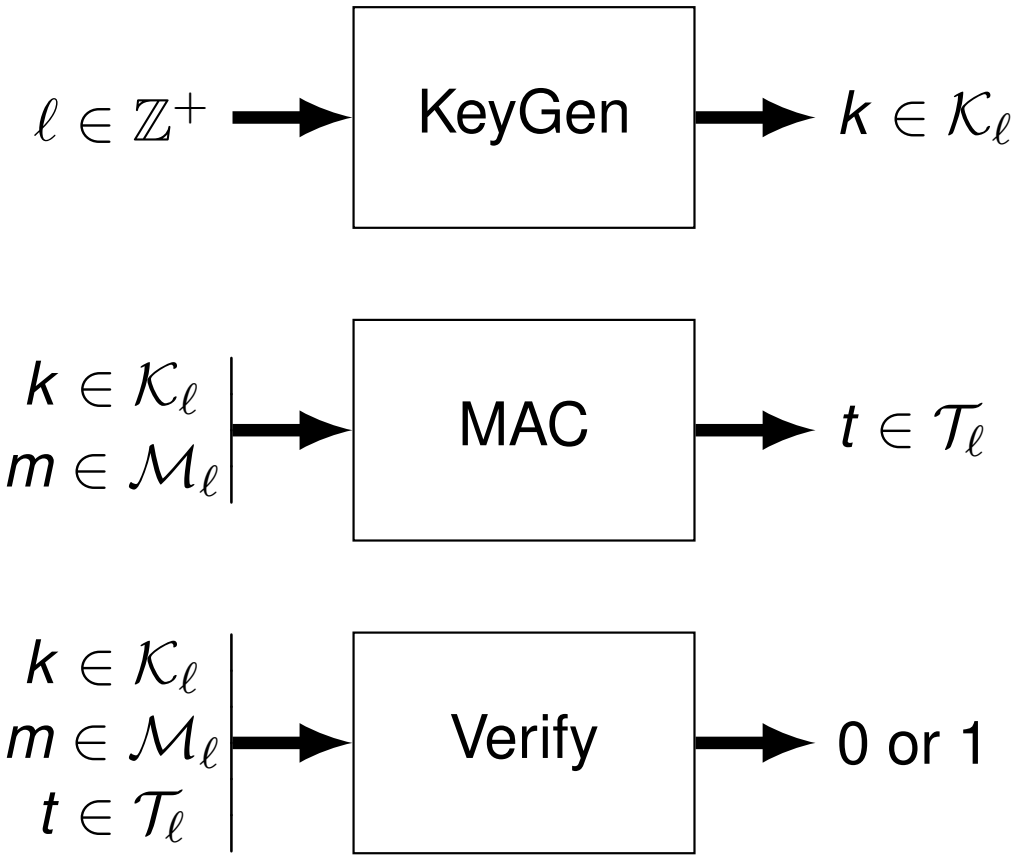
# Outline

# Message Authentication Codes: Syntax

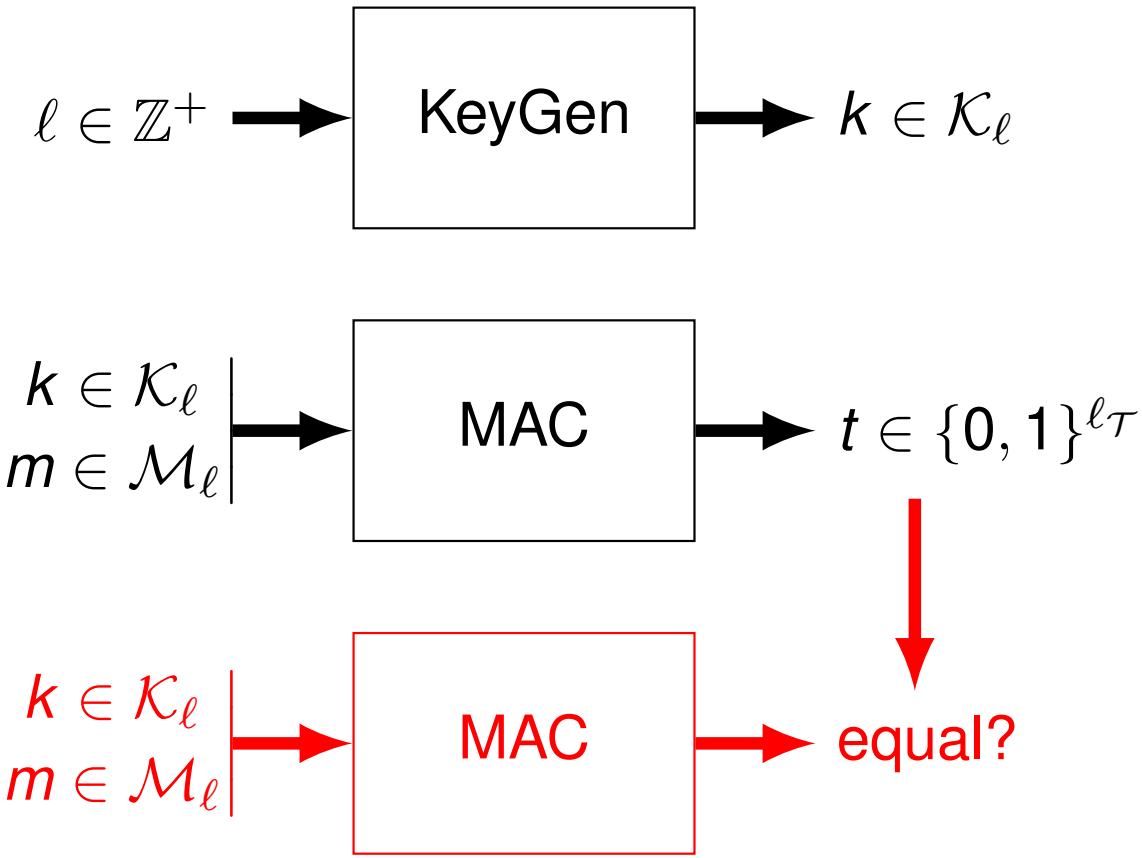


# Message Authentication Codes: Correctness




$$\forall m \in \mathcal{M}_\ell,\ \forall k \in \mathcal{K}_\ell, \quad \text{Verify}(k, m, \text{MAC}(k, m)) = 1$$

# Typical Verification Procedure




$$\text{Verify}(k, m, t) = 1 \;\Leftrightarrow\; t = \text{MAC}(k, m)$$

# MAC: Perfect Unforgeability

Informal definition:

"Impossible to find a new pair $(m', t')$ from $(m, t)$ without $k$"

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**Definition (Perfect $n$-Times Unforgeability)**

For a uniformly distributed $K \in \mathcal{K}_\ell$ and for any different $m_1, m_2, \ldots, m_n, m' \in \mathcal{M}_\ell$, **the random variables** $T_i = \mathrm{MAC}(K, m_i)$**,** $i = 1, \ldots, n$ **and** $T' = \mathrm{MAC}(K, m')$ **are independent.**

# Perfect One-Time MAC Construction

## Definition (Pairwise Independent Function Family)

$\mathcal{F} = \{f_k : \mathcal{X} \to \mathcal{Y}\}_{k \in \mathcal{K}}$ is a **pairwise independent** function family if for all different $x, x' \in \mathcal{X}$ the random variables $f_{U_\mathcal{K}}(x)$ and $f_{U_\mathcal{K}}(x')$ are independent, where $U_\mathcal{K}$ stands for a uniformly distributed random variable in $\mathcal{K}$.

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**Example:** $\mathcal{X} = \mathcal{Y} = \mathbb{Z}_q$, $\mathcal{K} = \mathbb{Z}_q^\times \times \mathbb{Z}_q$, for a $\ell$-bits long prime $q$, and $f_{a,b}(x) = ax + b \mod q$

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**Construction:**

KeyGen :
**output** $k \leftarrow \mathcal{K}$;

MAC$(k, m)$ :
**output** $f_k(m)$;

# MAC: Computational Unforgeability

$\Pi = (\text{KeyGen}, \text{MAC})$ for spaces $\mathcal{M}, \mathcal{C}, \mathcal{T}$.

**Experiment** Exp-MAC-EU-CMA$(\Pi, \mathcal{A}, \ell)$ :
$k \leftarrow \text{KeyGen}(\ell)$;
**table** $Q \leftarrow$ **empty_table**;
$(m', t') \leftarrow \mathcal{A}^{\mathcal{O}_{\text{MAC}}}(1^{\ell})$;
**if** $m' \notin Q$ **and** $t' = \text{MAC}(k, m')$ **output** 1; //$\mathcal{A}$ wins
**else output** 0;

**Oracle** $\mathcal{O}_{\text{MAC}}(m)$ :
**insert** $m$ **in** $Q$;
**output** $\text{MAC}(k, m)$;

## Definition (MAC-EU-CMA)

The MAC $\Pi$ is MAC-EU-CMA secure if for all Oracle PPTM, $\mathcal{A}$,

$$\Pr[\text{Exp-MAC-EU-CMA}(\Pi, \mathcal{A}, \ell) = 1] \in \textbf{negl}(\ell)$$

# Computationally Secure Constructions

Known efficient (constant length MAC) constructions based on

- hash functions (compression functions with fixed length output and hard to invert, find collisions, . . . )
- block ciphers
- . . .

# Codes and Cryptography

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END OF PART XII

# Proof (SE from PRP)

**Experiment**
Exp-PRP-Real$(\mathcal{F}, \mathcal{D}, \ell)$ :
$k \leftarrow \mathcal{K}_\ell$;
**output** $\mathcal{D}^{\text{Eval}}(1^\ell)$;

**Oracle** Eval$(m)$ :
**output** $f_k(m)$;

**Reduction:**
$q_{LR} \leftarrow 0$;
$b_* \leftarrow \{0,1\}$;
$b' \leftarrow \mathcal{A}^{\mathcal{O}_{\text{LR}}}(1^\ell)$;
**if** $b' = b_*$ **output** 1;
**else output** 0;

**Oracle** $\mathcal{O}_{\text{LR}}(m_0, m_1)$ :
**if** $q_{LR} > 0$
  **output** $\perp$;
**else**
  $q_{LR} \leftarrow 1$;
  **output** Eval$(m_{b_*})$;

**Experiment**
Exp-SE-LR$_1(\Pi_{\mathcal{F}}, \mathcal{A}, \ell)$ :
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  **output** $f_k(m_{b_*})$;

Perfect simulation!

# Proof (SE from PRP)

**Experiment**
Exp-PRP-Rand$(\mathcal{F}, \mathcal{D}, \ell)$ :
**table** $Q \leftarrow$ **empty_table**;
**output** $\mathcal{D}^{\text{Rand}}(1^\ell)$;

**Oracle** Rand$(m)$ :
**find** $c$ **for** $(m, c)$ **in** $Q$;
**if found output** $c$;
**else repeat**
  $c \leftarrow \{0,1\}^\ell$;
  **find** $c$ **for** $(m', c)$ **in** $Q$;
**while found**;
**insert** $(m, c)$ **in** $Q$;
**output** $c$;

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  **output** $f_k(m_{b_*})$;

$\mathcal{A}$'s view is independent of $b_*$

go back...

# Proof (SE from PRP with Padding)

**Experiment**
Exp-PRP-Real$(\mathcal{F}, \mathcal{D}, \ell)$ :
$k \leftarrow \mathcal{K}_\ell$;
**output** $\mathcal{D}^{\text{Eval}}(1^\ell)$;

**Oracle** Eval$(x)$ :
**output** $f_k(x)$;

**Reduction:**
$b_* \leftarrow \{0,1\}$;
$b' \leftarrow \mathcal{A}^{\mathcal{O}_{\text{LR}}}(1^\ell)$;
**if** $b' = b_*$ **output** 1;
**else output** 0;

**Oracle** $\mathcal{O}_{\text{LR}}(m_0, m_1)$ :
$r \leftarrow \{0,1\}^\kappa$;
**output** Eval$(m_{b_*} \| r)$;

**Experiment**
Exp-SE-LR$(\Pi_{\mathcal{F}}, \mathcal{A}, \ell)$ :
$k \leftarrow \mathcal{K}_\ell$;
$b_* \leftarrow \{0,1\}$;
$b' \leftarrow \mathcal{A}^{\mathcal{O}_{\text{LR}}}(1^\ell)$;
**if** $b' = b_*$ **output** 1;
**else output** 0;

**Oracle** $\mathcal{O}_{\text{LR}}(m_0, m_1)$ :
$r \leftarrow \{0,1\}^\kappa$;
**output** $f_k(m_{b_*} \| r)$;

Perfect simulation!

# Proof (SE from PRP with Padding)

**Experiment**
Exp-PRP-Rand$(\mathcal{F}, \mathcal{D}, \ell)$ :
**table** $Q \leftarrow$ **empty_table**;
**output** $\mathcal{D}^{\text{Rand}}(1^\ell)$;

**Oracle** Rand$(x)$ :
**find** $c$ **for** $(x, c)$ **in** $Q$;
**if found output** $t$;
**else repeat**
  $c \leftarrow \{0,1\}^\ell$;
  **find** $c$ **for** $(x', c)$ **in** $Q$;
**while found**;
**insert** $(x, c)$ **in** $Q$;
**output** $c$;

**Reduction:**
$b_* \leftarrow \{0,1\}$;
$b' \leftarrow \mathcal{A}^{\mathcal{O}_{\text{LR}}}(1^\ell)$;
**if** $b' = b_*$ **output** 1;
**else output** 0;

**Oracle** $\mathcal{O}_{\text{LR}}(m_0, m_1)$ :
$r \leftarrow \{0,1\}^\kappa$;
**output** Rand$(m_{b_*} \| r)$;

**Experiment**
Exp-SE-LR$(\Pi_{\mathcal{F}}, \mathcal{A}, \ell)$ :
$k \leftarrow \mathcal{K}_\ell$;
$b_* \leftarrow \{0,1\}$;
$b' \leftarrow \mathcal{A}^{\mathcal{O}_{\text{LR}}}(1^\ell)$;
**if** $b' = b_*$ **output** 1;
**else output** 0;

**Oracle** $\mathcal{O}_{\text{LR}}(m_0, m_1)$ :
$r \leftarrow \{0,1\}^\kappa$;
**output** $f_k(m_{b_*} \| r)$;

$\mathcal{A}$'s view is (nearly) independent of $b_*$

go back...