Codes and Cryptography

Jorge L. Villar

MAMME, Fall 2015

PART XI

Outline

Defining a Security Notion

Defining security for a particular system requires:

- Defining the functionality of the system
- Defining the capabilities of the adversary
- Defining the goal of the adversary

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- Defining the goal of the adversary

The latter two can be captured by

- a random experiment (game) between a Challenger and the Adversary
- a special outcome indicating success of the Adversary
- a statement about the probability of that outcome

Example 1: One-Way Security

Assume that $\Pi = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ is a symmetric encryption scheme for the spaces $\mathcal{M}$, $\mathcal{C}$, $\mathcal{K}$ and security parameter ℓ.

Experiment $\mathsf{Exp\text{-}SE\text{-}OW}(\Pi, \mathcal{A}, \ell)$:
$k \leftarrow \mathsf{KeyGen}(\ell)$;
$m_* \leftarrow \mathcal{M}_\ell$;
$c_* \leftarrow \mathsf{Enc}(k, m_*)$;
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if $m' = m_*$ **output** 1; //$\mathcal{A}$ wins
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The security statement is

Definition (SE-OW)

The symmetric encryption scheme Π is SE-OW secure if for all Probabilistic Polynomial-Time Turing Machine (PPTM), $\mathcal{A}$,

$$\Pr[\text{Exp-SE-OW}(\Pi, \mathcal{A}, \ell) = 1] \in \textbf{negl}(\ell)$$

Example 2: Stronger Attacks

In some practical scenarios, an adversary has access to some pairs plaintext/ciphertext for the target key.

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Oracle $\mathcal{O}_{\text{Enc}}(m)$:
output $\text{Enc}(k, m)$;

Oracle $\mathcal{O}_{\text{Dec}}(c)$:
if $c = c_*$ **output** $\perp$; //Illegal oracle query
else output $\text{Dec}(k, c)$;

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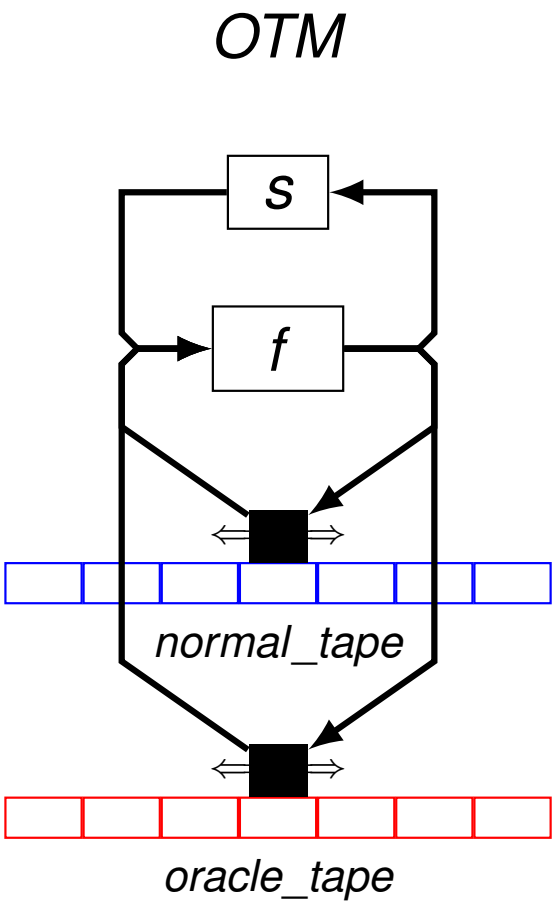
The number of queries q_{Enc} and q_{Dec} can be considered as additional security parameters

Oracle Turing Machine



Special state: '`oracle_query`'

The OTM enters in a waiting state until some external entity (not necessarily a Turing Machine) replaces the information in the oracle tape, in unit time.

NOTATION: $OTM^{\mathcal{O}}$

The oracle tape is used as a communication tape. Interactive Turing Machines can be defined following the same idea.

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$b' \leftarrow \mathcal{A}^{\mathcal{O}_{\mathrm{LR}}}(1^\ell)$;
if $b' = b_*$ **output** 1; //$\mathcal{A}$ wins
else output 0;

Oracle $\mathcal{O}_{\mathrm{LR}}(m_0, m_1)$:
if length(m_0) $\neq$ length(m_1) **output** $\perp$; //Illegal oracle query
output Enc(k, m_{b_*});

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The other notions SE-LR-CPA and SE-LR-CCA are defined accordingly

Outline

Translating Languages

Reduction: An efficient transformation $T : \{0,1\}^* \to \{0,1\}^*$ that maps a language L into another language L', and also maps $\{0,1\}^* \setminus L$ into $\{0,1\}^* \setminus L'$.

NOTATION: $L \Rightarrow_{PP} L'$ or "L reduces to L'"

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A language L is PP-reducible to another language L' if there exists a PPTM T and a integer-valued function $q \in$ **poly** such that $T(\{0,1\}^\ell) \subseteq \{0,1\}^{q(\ell)}$, $T(L) \subseteq L'$ and $T(\{0,1\}^* \setminus L) \subseteq \{0,1\}^* \setminus L'$

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Theorem

$L \notin \mathcal{BPP}$ and $L \Rightarrow_{PP} L'$ implies $L' \notin \mathcal{BPP}$

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Constructive proof: Explicitly (and efficiently) build a PPTM solving P from another PPTM solving P'

Reducing Computational Problems

Constructive proofs for the statement $P \Rightarrow_{PP} P'$:
Give a PPTM $\mathcal{R}$ that transforms (the description of) any PPTM $\mathcal{A}'$ solving a random instance of P' into (the description of) another PPTM $\mathcal{A} = \mathcal{R}[\mathcal{A}']$ solving P such that

$$\mathrm{Succ}_{P',\mathcal{A}'}(\ell) > \mathbf{negl}(\ell) \quad \Rightarrow \quad \mathrm{Succ}_{P,\mathcal{R}[\mathcal{A}']}(\ell) > \mathbf{negl}(\ell)$$

where $\mathrm{Succ}_{P,\mathcal{A}}(\ell)$ is $\Pr[\mathcal{A}(x) \in \mathrm{sol}(x) \ : \ x \leftarrow P_\ell]$ for search problems, and

$$\left|\Pr[\mathcal{A}(x) = 1 \ : \ x \leftarrow L_P \cap \{0,1\}^\ell] - \Pr[\mathcal{A}(x) = 1 \ : \ x \leftarrow \{0,1\}^\ell \setminus L_P]\right|$$

for decision problems

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A typical reduction: Black-Box with a single call to $\mathcal{A}'$:

- $\mathcal{R}[\mathcal{A}']$ transforms its input $x \in P$ into $x' \in P'$
- $\mathcal{R}[\mathcal{A}']$ runs $\mathcal{A}'$ with input x'
- $\mathcal{R}[\mathcal{A}']$ computes its output from the output of $\mathcal{A}'$

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For small $\mathrm{Succ}_{P,\mathcal{A}'}(\ell)$

$$\mathrm{Succ}_{P,\mathcal{R}[\mathcal{A}']}(\ell) \approx \sqrt{\frac{2n}{\pi}}\,\mathrm{Succ}_{P,\mathcal{A}'}(\ell)$$

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For ('checkable') search problems and small $\mathrm{Succ}_{P,\mathcal{A}'}(\ell)$

$$\mathrm{Succ}_{P,\mathcal{R}[\mathcal{A}']}(\ell) \approx n\,\mathrm{Succ}_{P,\mathcal{A}'}(\ell)$$

and the meaningful quantity for comparisons is probability/time

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Using T as a self-reduction $\mathcal{R}_T$,

$$\mathcal{A}(x) = \mathcal{R}_T[\mathcal{A}'](x) = \mathcal{A}'(T(x))$$

proves that solving a random instance of P is not easier than (thus, equivalent to) solving **all** instances in P.

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For a random self-reducible problem **average hardness is equivalent to worst-case hardness**

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Reductions between security notions show implications, or relative hardness, e.g., ▸ details...

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Thus, $\mathcal{R}$ **simulates** any oracle given in SEC2 for $\mathcal{A}_2$, but it can **use** the oracles given in SEC1.

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Security proofs by reduction: A reduction of a computational problem family P to the problem of breaking a security notion SEC for a cryptosystem Π, proves security of Π under the assumption that P is hard

$$P \Rightarrow \mathrm{SEC}\langle\Pi\rangle$$

It reads "if someone breaks Π, he also solves P"

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. . . but some reductions are not meaningful in practice. . .

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More precisely, "there exists $\mathcal{R}$ such that if $\mathcal{A}$ breaks Π in time t_1 with probability/advantage $\varepsilon_1 > \mathbf{negl}(\ell)$, then $\mathcal{R}[\mathcal{A}]$ solves P in time t_2 with probability/advantage $\varepsilon_2 > \mathbf{negl}(\ell)$"

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- If $t_2 \approx t_1$ but $\varepsilon_2/\varepsilon_1 \to 0$ as $\ell \to \infty$, $\mathcal{R}$ is almost **not tight**

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- If $t_2 \gg t_1$, compare the ratios ε_1/t_1 and ε_2/t_2

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MAMME, Fall 2015

END OF PART XI

A Sample Reduction: SE-LR-CPA $\Rightarrow$ SE-OW-CPA

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$k \leftarrow \text{KeyGen}(\ell)$;
$m_* \leftarrow \mathcal{M}_\ell$;
$c_* \leftarrow \text{Enc}(k, m_*)$;
$m' \leftarrow \mathcal{A}^{\mathcal{O}_{\text{Enc}}}(1^\ell, c_*)$;
if $m' = m_*$
  output 1;
else output 0;

Oracle $\mathcal{O}_{\text{Enc}}(m)$:
output $\text{Enc}(k, m)$;

A Sample Reduction: SE-LR-CPA $\Rightarrow$ SE-OW-CPA

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Oracle $\mathcal{O}_{\text{LR}}(m_0, m_1)$:
if $|m_0| \neq |m_1|$
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