Cryptology

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FME, UPC, Fall 2024

Outline

1 Stream Ciphers

2 Block Ciphers

3 Block Chaining Modes

4 Key Management

5 Message Authentication Codes

Construction Strategies

- **Stream ciphers:** For arbitrarily long messages (e.g., data streams).
  A key $k \in \mathcal{K}$ and a nonce r are expanded to a **pseudorandom** stream, $g(r, k)$, used as a one-time pad.

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- **Block ciphers:** For messages with a fixed length.
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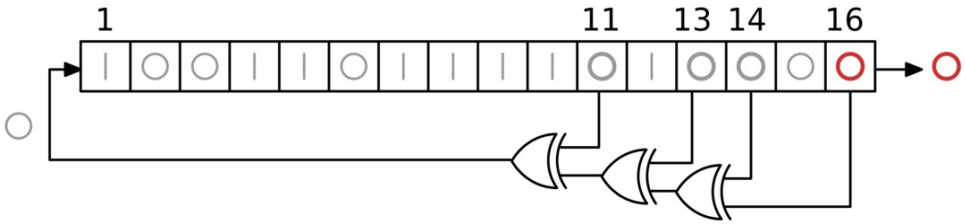
with a **chaining mode** can encrypt arbitrarily long messages

Examples of stream ciphers

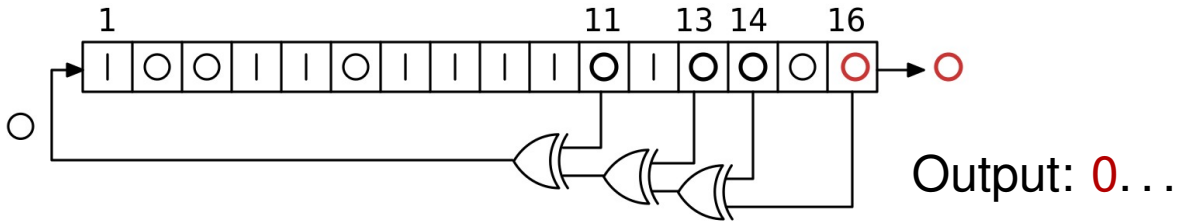
- **Linear Feedback Shift Registers.** Ultra fast but insecure.



(image based on https://commons.wikimedia.org/w/index.php?curid=60225898)

- **Blum-Blum-Shub generator.** Very slow. Provably secure.
- **Non-Linear Feedback Shift Registers.** Very fast. Widely used (GSM, Bluetooth, . . .). In general, they lead to weak symmetric encryption schemes.
- **Other designs:** RC4 (obsolete), chacha, ...

Linear Feedback Shift Registers (LFSR)



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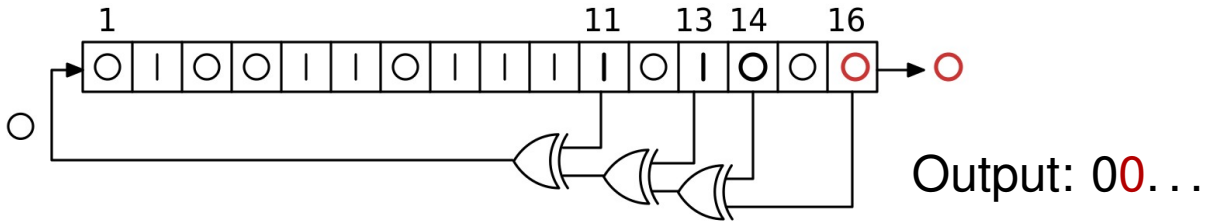

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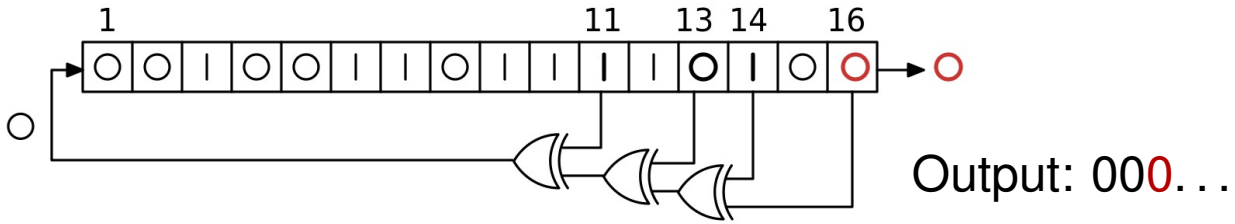

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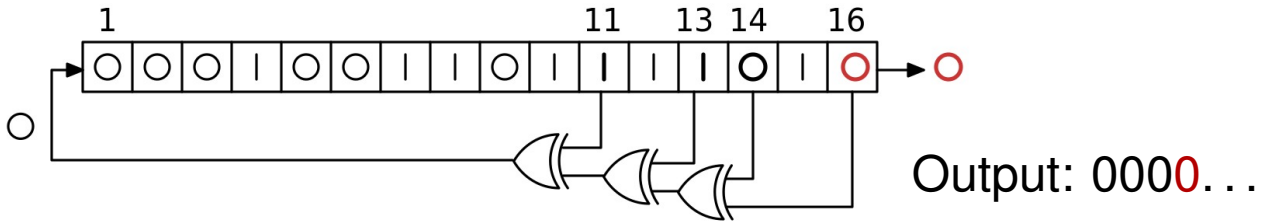

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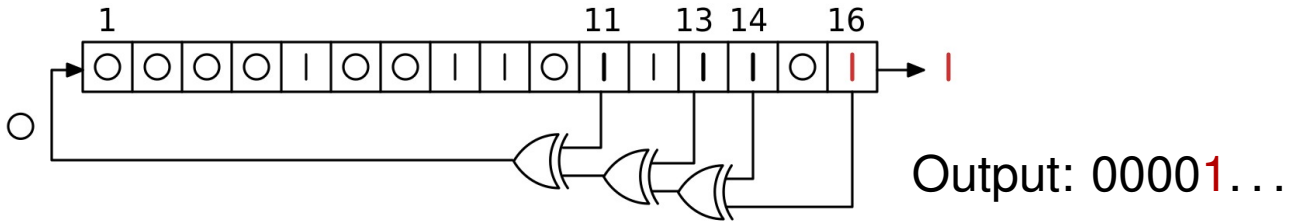

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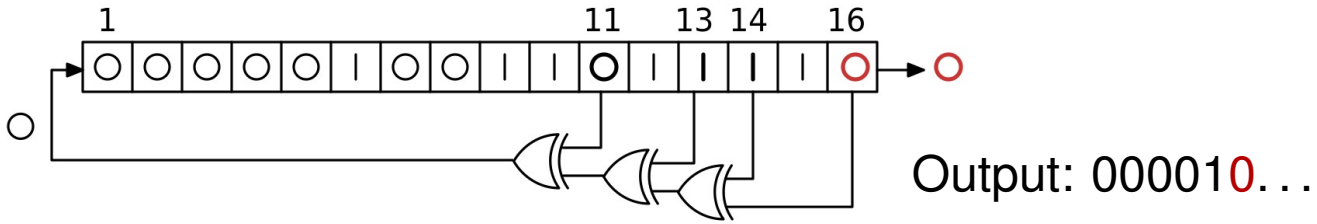

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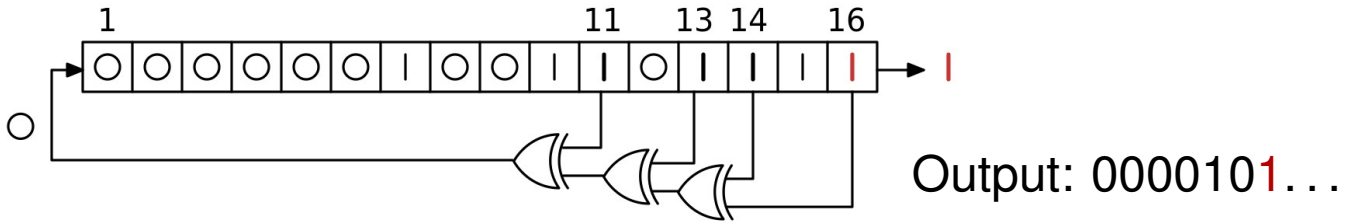

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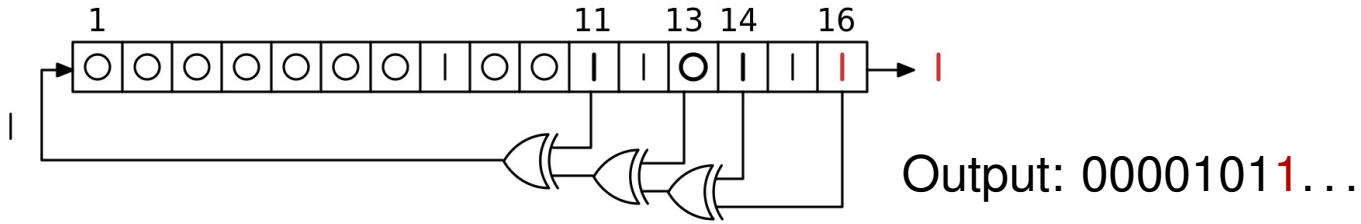

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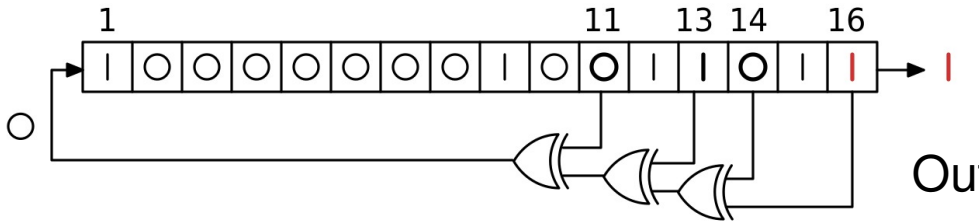

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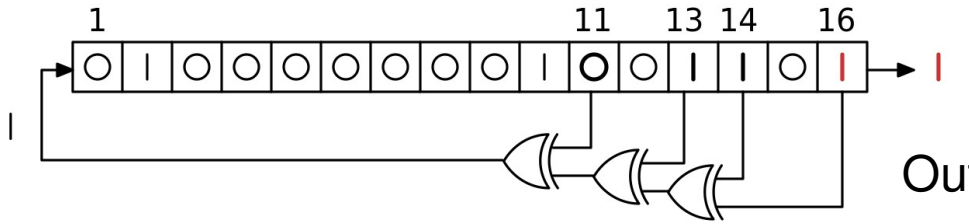

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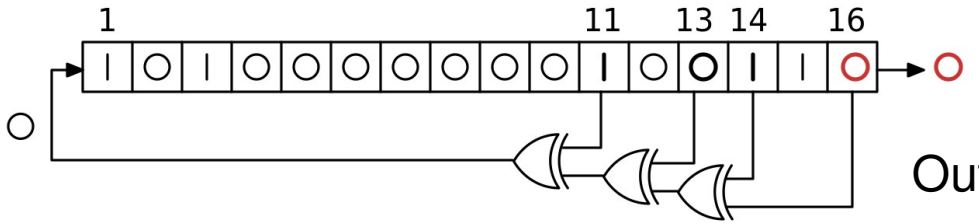

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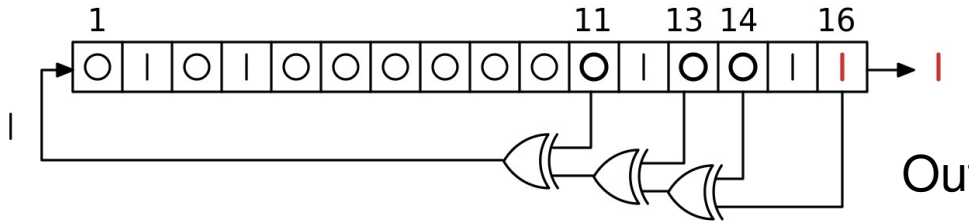

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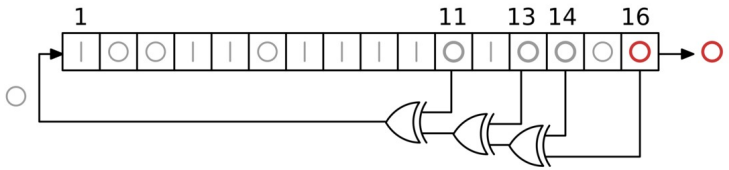
Output equation: $y_{n-16} \oplus y_{n-14} \oplus y_{n-13} \oplus y_{n-11} \oplus y_n = 0$

LFSR (II)

$$y_0 = x_{16}$$
$$y_1 = x_{15}$$
$$\vdots$$
$$y_{15} = x_1$$



$$y_{16} = y_0 \oplus y_2 \oplus y_3 \oplus y_5 \quad \Rightarrow \quad y_0 \oplus y_2 \oplus y_3 \oplus y_5 \oplus y_{16} = 0$$
$$y_{17} = y_1 \oplus y_3 \oplus y_4 \oplus y_6 \quad \Rightarrow \quad y_1 \oplus y_3 \oplus y_4 \oplus y_6 \oplus y_{17} = 0$$
$$y_{18} = y_2 \oplus y_4 \oplus y_5 \oplus y_7 \quad \Rightarrow \quad y_2 \oplus y_4 \oplus y_5 \oplus y_7 \oplus y_{18} = 0$$
$$\vdots$$

Characteristic polynomial: $P(z) = \underbrace{z^{16} + z^{14} + z^{13} + z^{11}}_{\text{LFSR wires}} + 1$

In general, in $\mathbb{F}_2[z]$:

$$\deg(P(z)(\underbrace{y_0 + y_1 z + y_2 z^2 + \ldots}_{Y(z)})) < \deg(P(z)) = m$$

LFSR (II)

Proposition

The output sequence of an LFSR is ultimately periodic.

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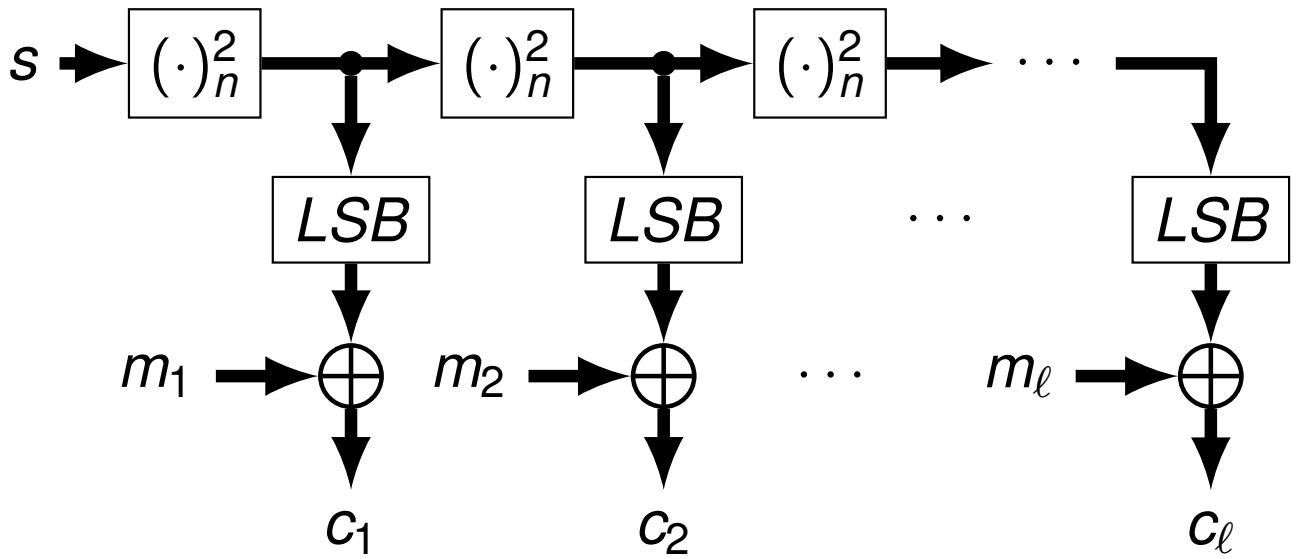
Proposition

From any output subsequence of length $2m$ the structure of a LFSR of size $\leq m$ producing the sequence can be efficiently determined.

LFSR do not yield secure stream ciphers!

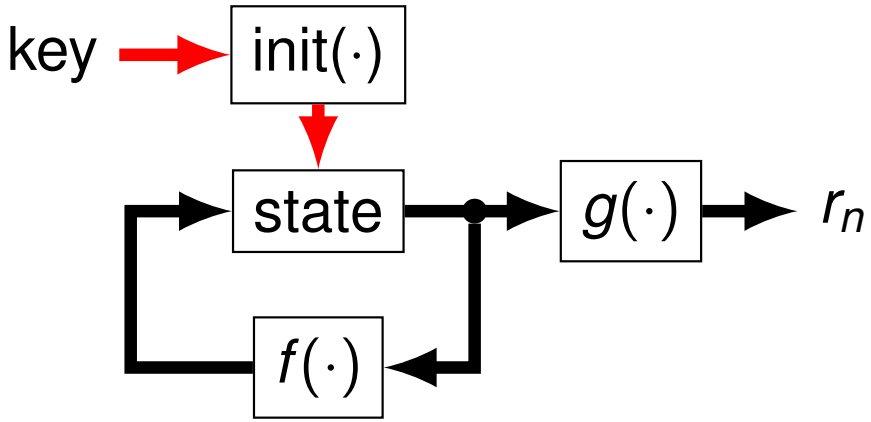
Stream Cipher from Blum-Blum-Shub Generator



$n = pq$, $p = 2p' + 1$, $q = 2q' + 1$
p, q, p', q' primes (512 to 1024 bits).

RC4 Stream Cipher

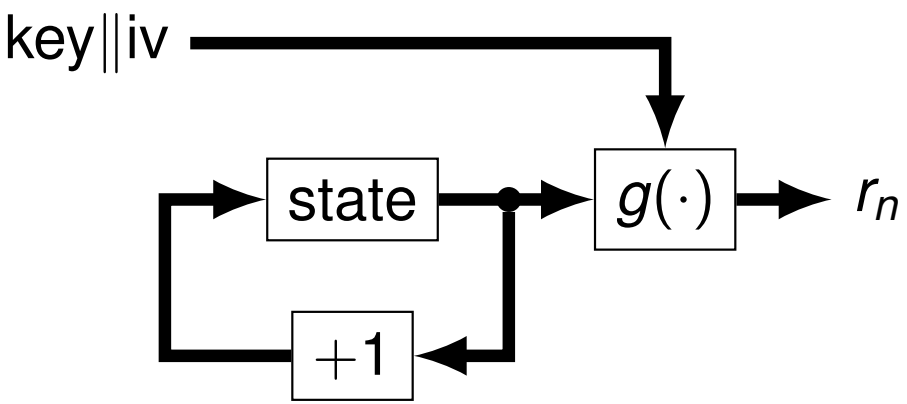
- Internal state: a permutation s of $\{0, \ldots, 255\}$ and two counters i, j.



- initial state: computed from the key.
- Every message byte m_n is XORed with the output byte r_n.
- The key is too short (from 40 bits to 128 bits).
- No specified way to use an initialization value (iv).
- Some real systems using RC4 have been attacked.

Chacha Stream Cipher

- Internal state: just a counter i.
- It works with 512 bit words.



- initial state: $i = 0$.
- Every message word m_n is XORed with the output word r_n.

- key has 256 bits and the iv has 64 bits.
- fully parallelizable (random access to words)
- No successful attack is currently known.

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Practical Construction of Block Ciphers

Heuristic constructions based on:

- **Confusion:** every bit in the ciphertext must depend on several bits of the key.
- **Diffusion:** flipping a single plaintext bit must change half of the ciphertext bits.
- **Iteration:** the encryption procedure consists of a given number of rounds.

Structure:

- **Key expansion:** a set of round keys is generated from the original key
- **Mixing** of permutation (shuffling, shifting. . .), substitution (S-boxes) and XORing the subkeys in each round.

Example: DES

Key size: 56 bits

Message and ciphertext block size: 64 bits

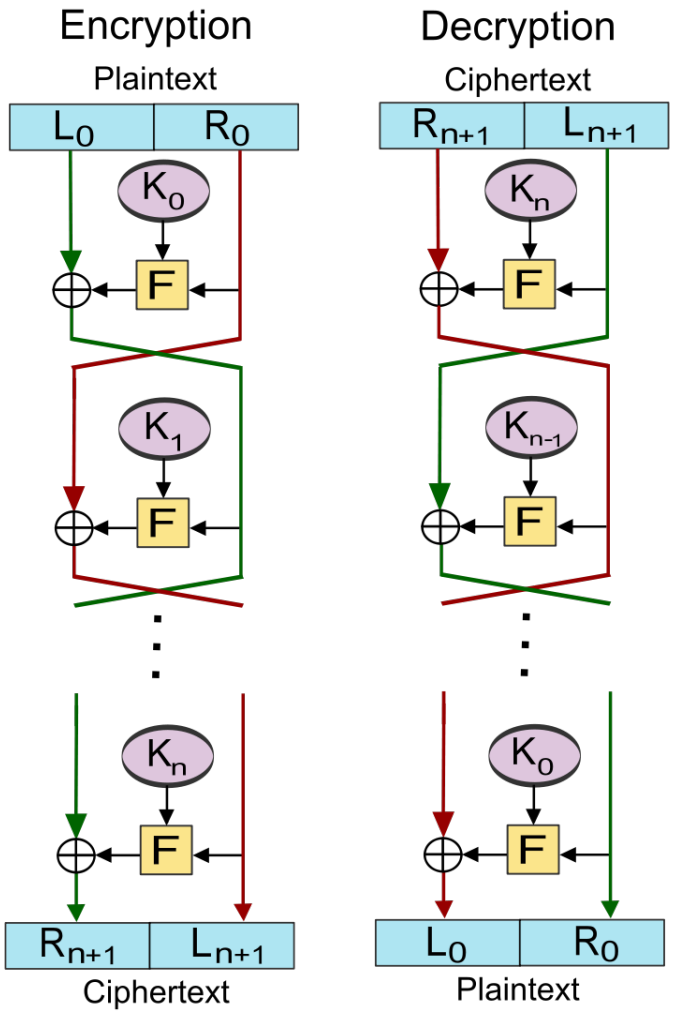
Number of encryption rounds: 16

Key expansion: from 56 bits to 768 bits (16 subkeys, each one of 48 bits)

Encryption rounds: Feistel Network

- Start with the 64 bit message block
- In each iteration, divide the intermediate block into two halves (L_i, R_i)
- Apply a transformation to compute next block $(L_{i+1}, R_{i+1}) = (R_i, L_i \oplus F(k_i, R_i))$
- The resulting ciphertext block is (R_{16}, L_{16}).

Feistel Network



Example: DES (II)

The function F combines:

- Expands the 32 bit half-block into 48 bits, and XORs the round subkey.
- Divides the 48 bits into 8 6-bit words.
- Transforms each word into a 4 bit word with a table (S-box).
- Glues the 4 bit words to obtain a 32 bit half-block.
- Applies a fixed prermutation to the 32 bits.

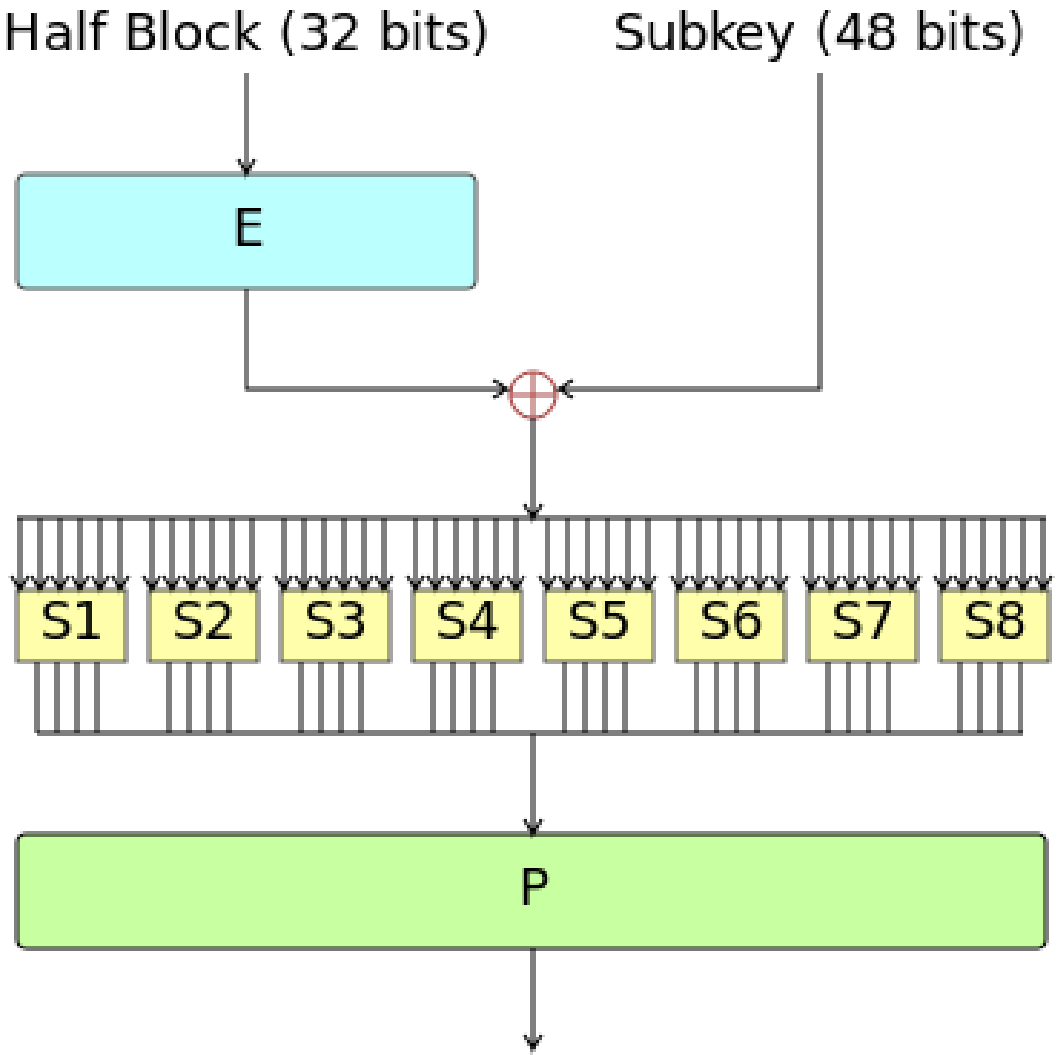
3DES combines three independent keys (168 bits in total) and it is still considered secure.

$3DES_{k_1,k_2,k_3}(m) = DES_{k_3} \circ DES_{k_2}^{-1} \circ DES_{k_1}(m)$

More details in

http://en.wikipedia.org/wiki/Data_Encryption_Standard

Example: DES Round Function



Example: AES256

Key size: 256 bits

Message and ciphertext block size: 128 bits (4×4 byte matrix)

Number of encryption rounds: 14

Key expansion: from 256 bits to 1920 bits (15 subkeys, each one a 4×4 matrix of bytes) in an iterative procedure (it adds 4 new bytes in each iteration)

Encryption round:

- Apply a substitution box to each byte
- Perform rotation operations to rows
- Perform linear transformation to columns
- XOR with the round key

More details in
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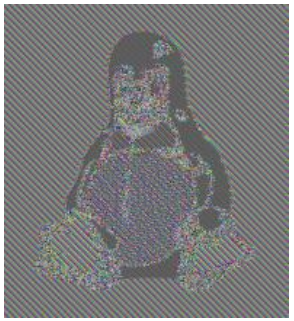

(from Wikipedia)

Equal message blocks result in equal ciphertext blocks!

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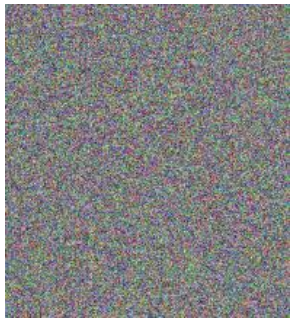
CBC (Cipher Block Chaining):

$c_0 = iv$

$c_i = \mathrm{Enc}(k, m_i \oplus c_{i-1})$



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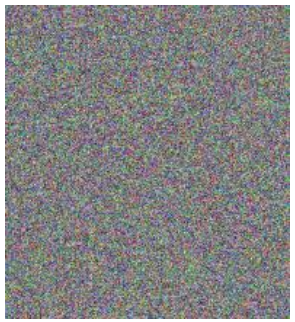

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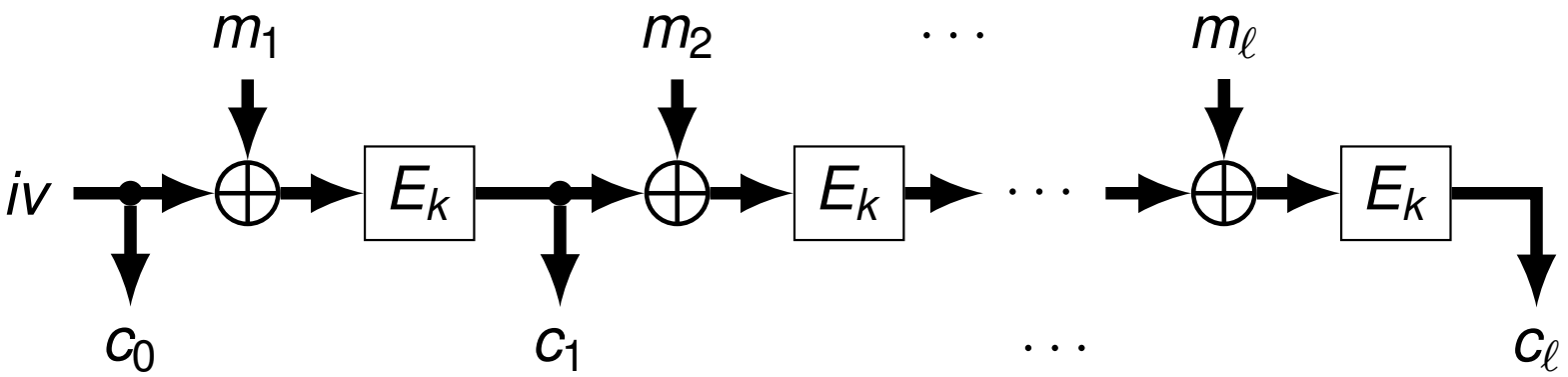
The ciphertext has an extra random block c_0, the "initialization vector"

CBC Mode

Comp. . .



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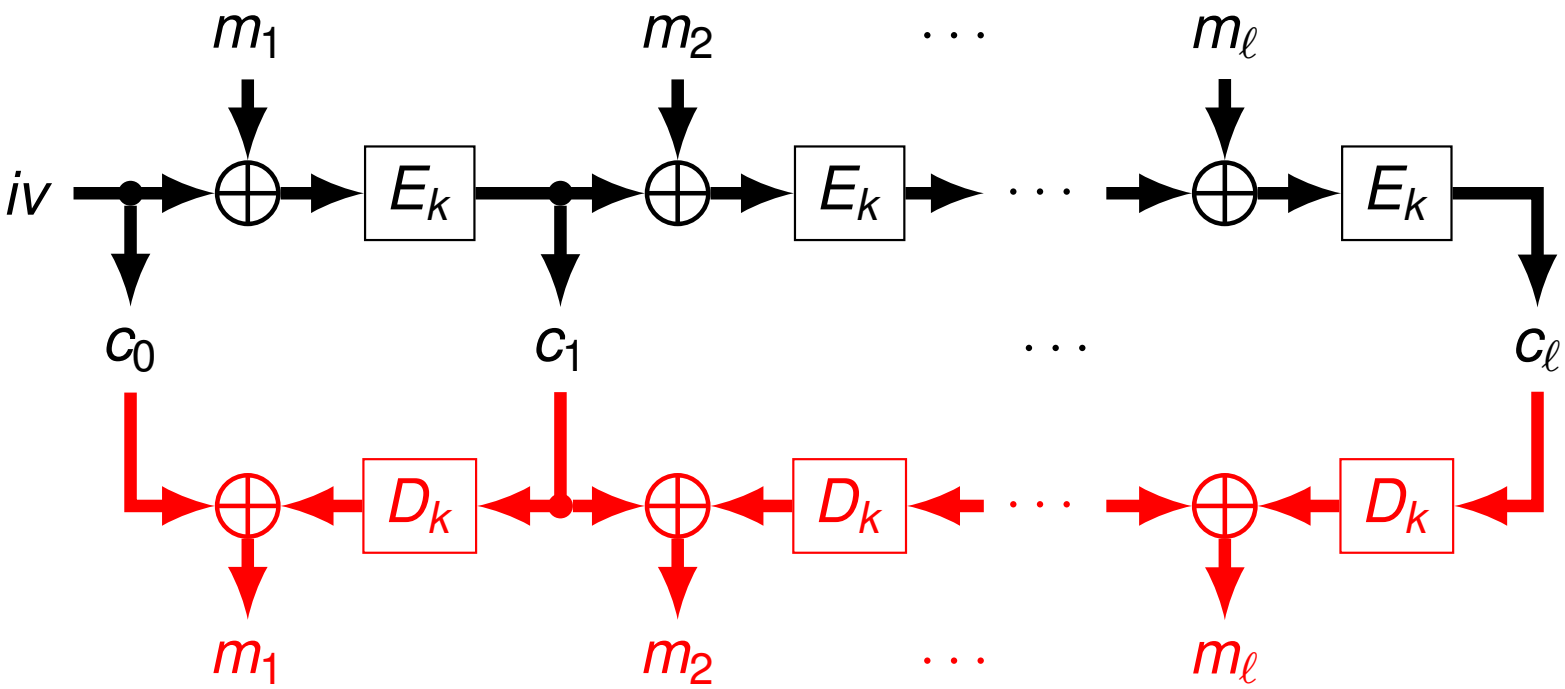

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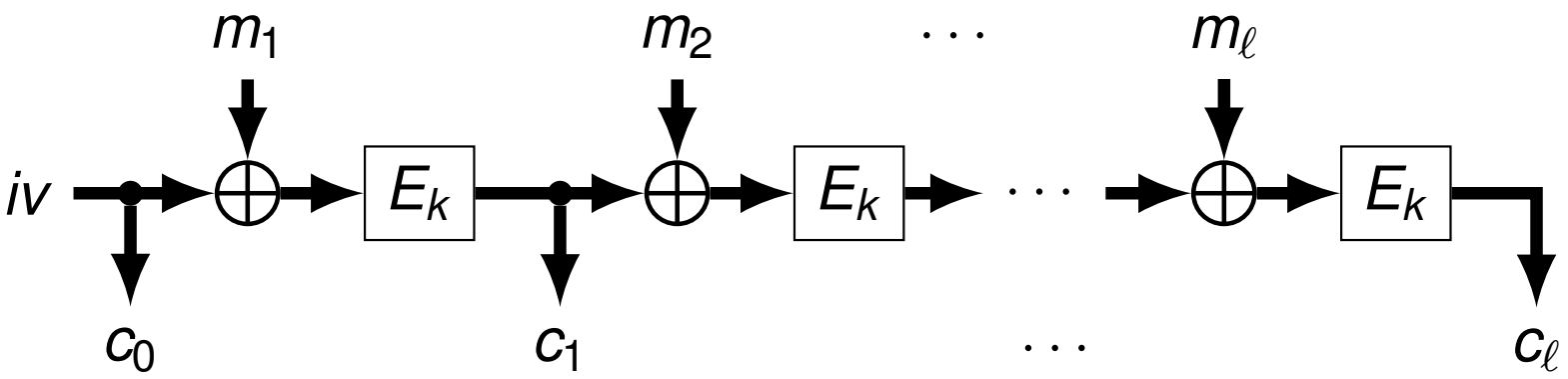

$$c_i = c_j \quad \Rightarrow \quad m_i \oplus c_{i-1} = m_j \oplus c_{j-1} \quad \Rightarrow \quad m_i \oplus m_j \text{ revealed!}$$

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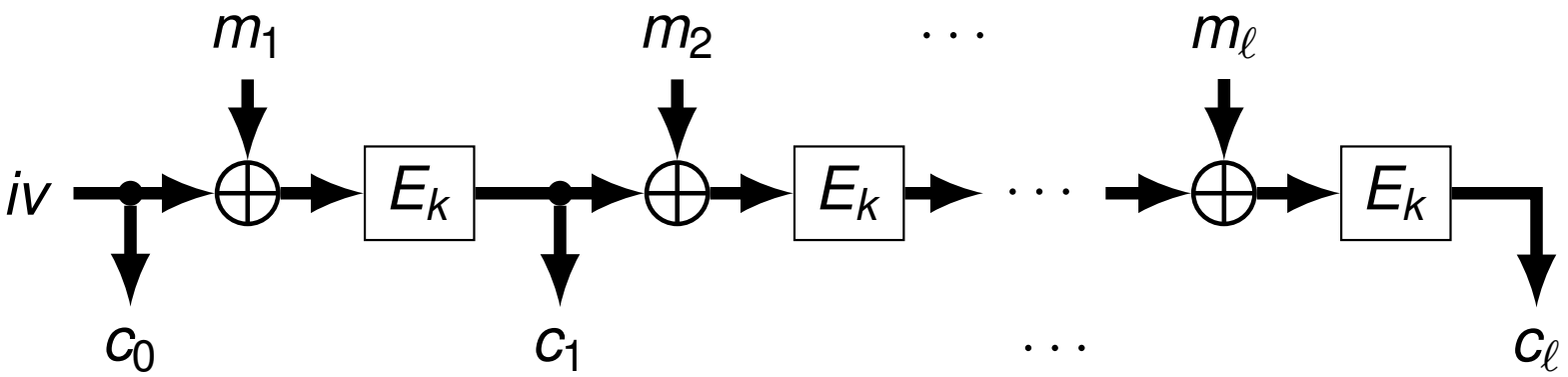

$c_i = c_j \quad \Rightarrow \quad m_i \oplus c_{i-1} = m_j \oplus c_{j-1} \quad \Rightarrow \quad m_i \oplus m_j$ revealed!

Collision probability must be small

Other Block Chaining Modes

- CFB (Cipher Feedback) operation mode:

  $c_0 = iv$

  $c_i = \text{Enc}(k, c_{i-1}) \oplus m_i$

- OFB (Output Feedback) operation mode:

  $r_0 = iv$

  $r_i = \text{Enc}(k, r_{i-1})$

  $c_i = m_i \oplus r_i$

- CTR (Counter) operation mode:

  $r_i = \text{Enc}(k, iv + i)$

  $c_i = m_i \oplus r_i$

CFB Mode

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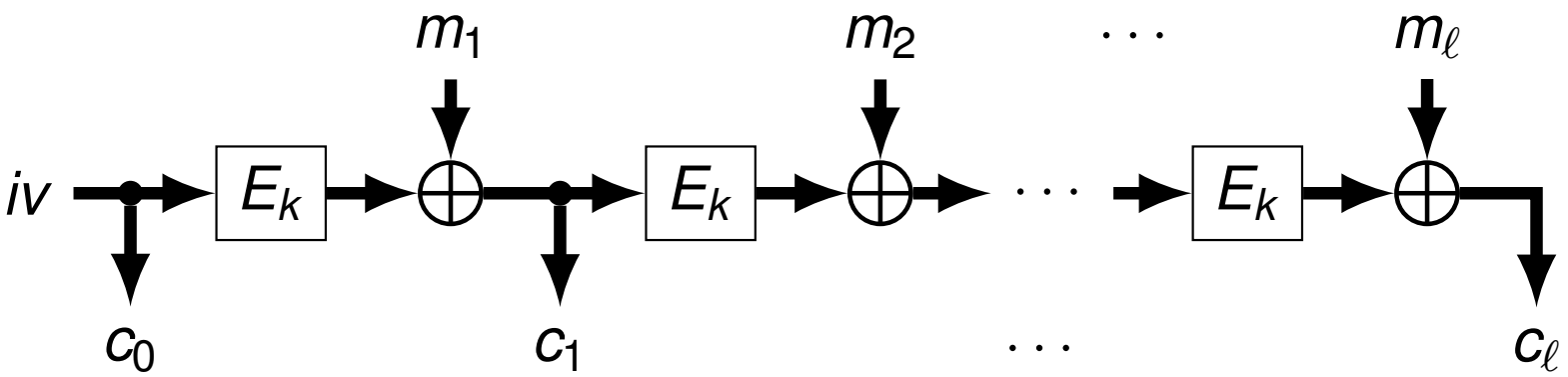
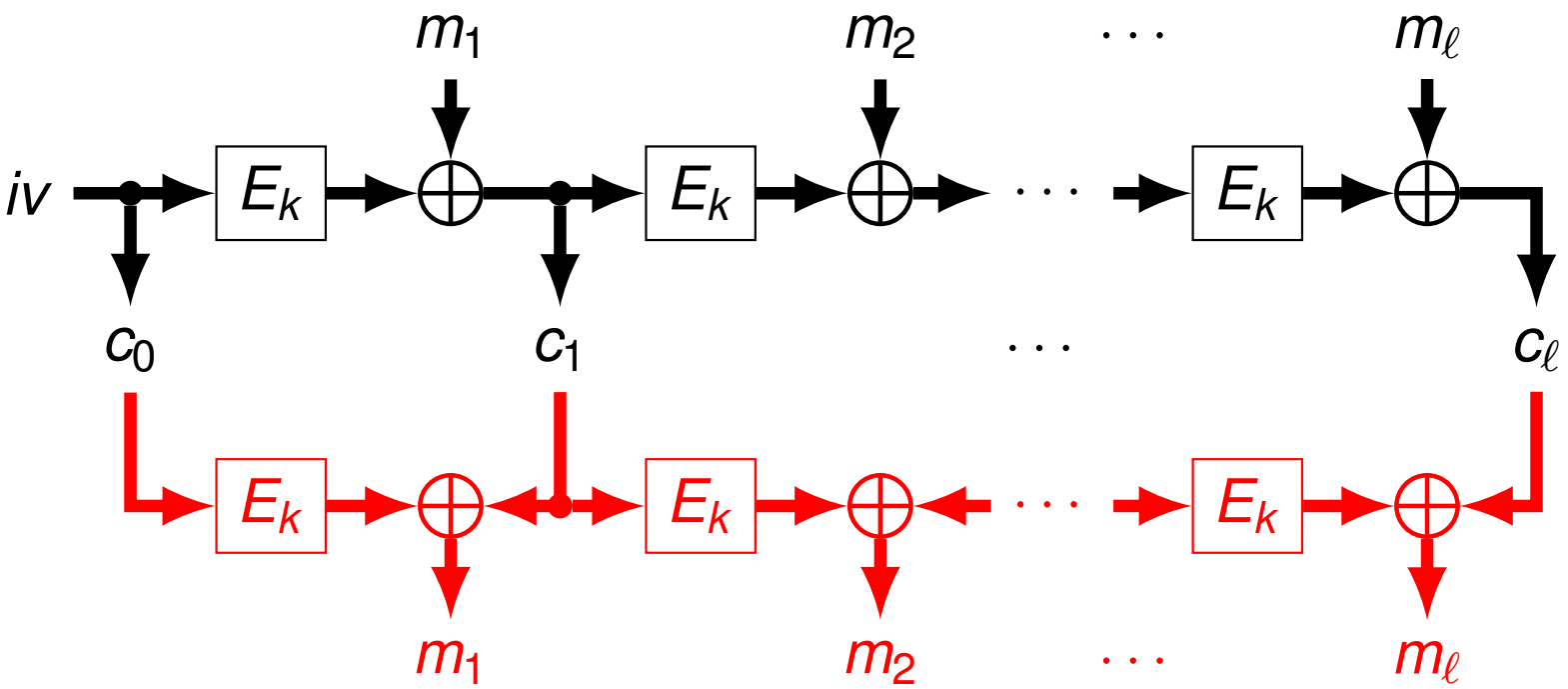


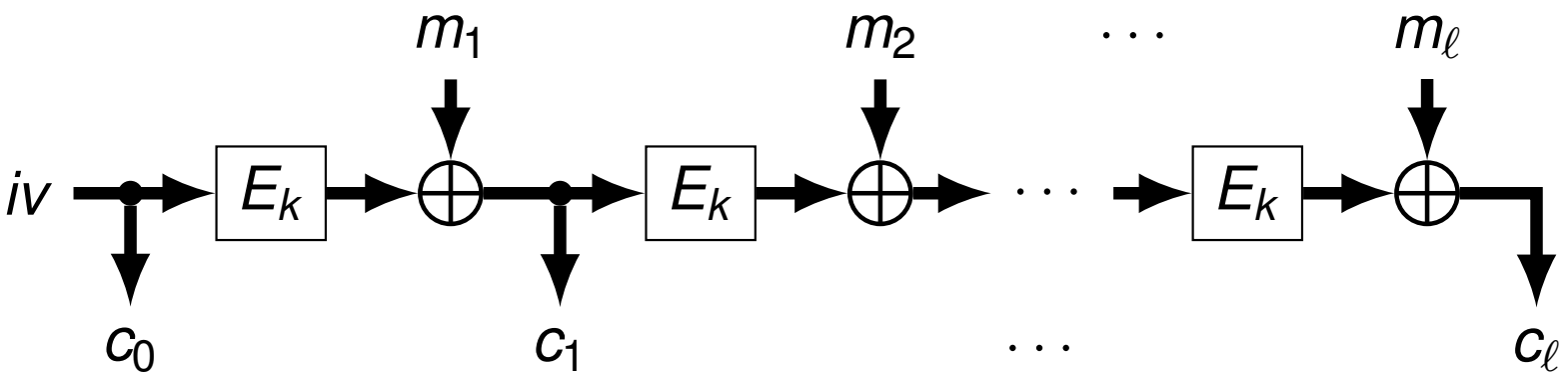
CFB Mode

Comp. . .



CFB Mode

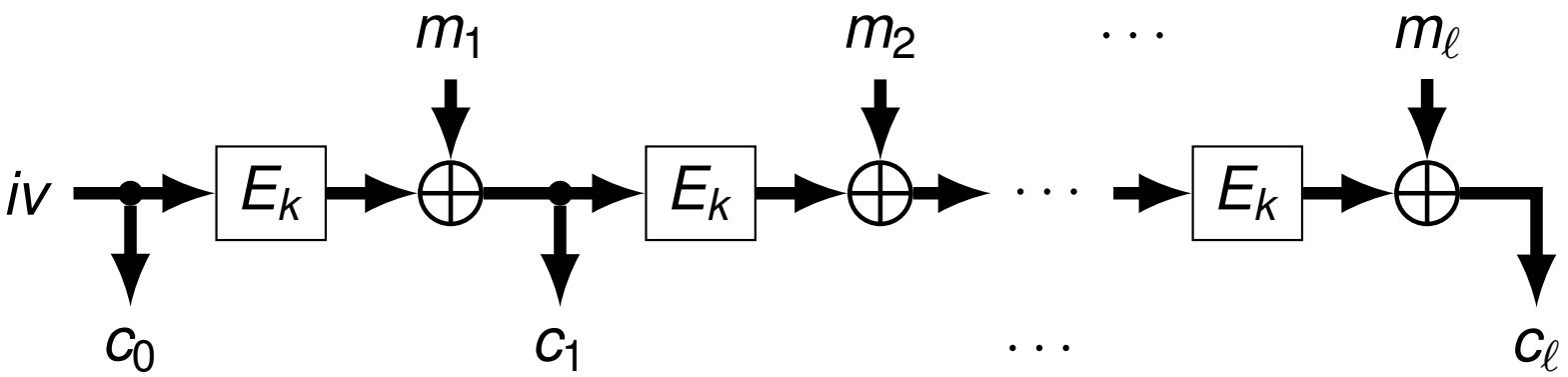
Comp...



$$c_{i-1} = c_{j-1} \quad \Rightarrow \quad m_i \oplus c_i = m_j \oplus c_j \quad \Rightarrow \quad m_i \oplus m_j \text{ revealed!}$$

CFB Mode

Comp...

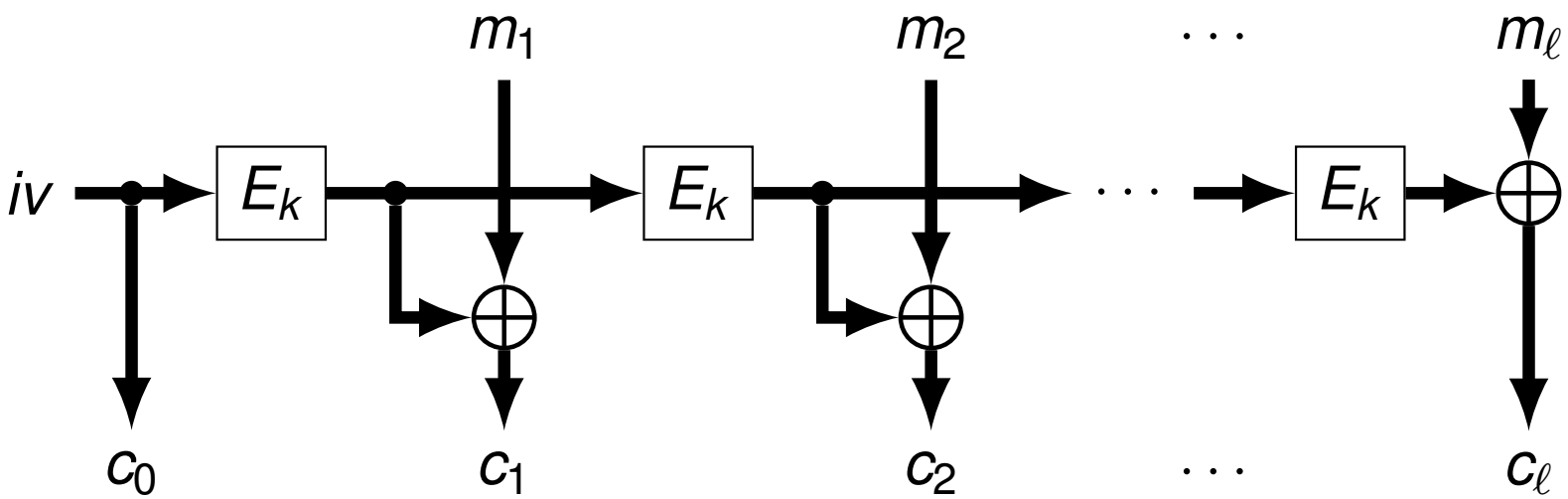


$c_{i-1} = c_{j-1} \quad \Rightarrow \quad m_i \oplus c_i = m_j \oplus c_j \quad \Rightarrow \quad m_i \oplus m_j$ revealed!

Collision probability must be small

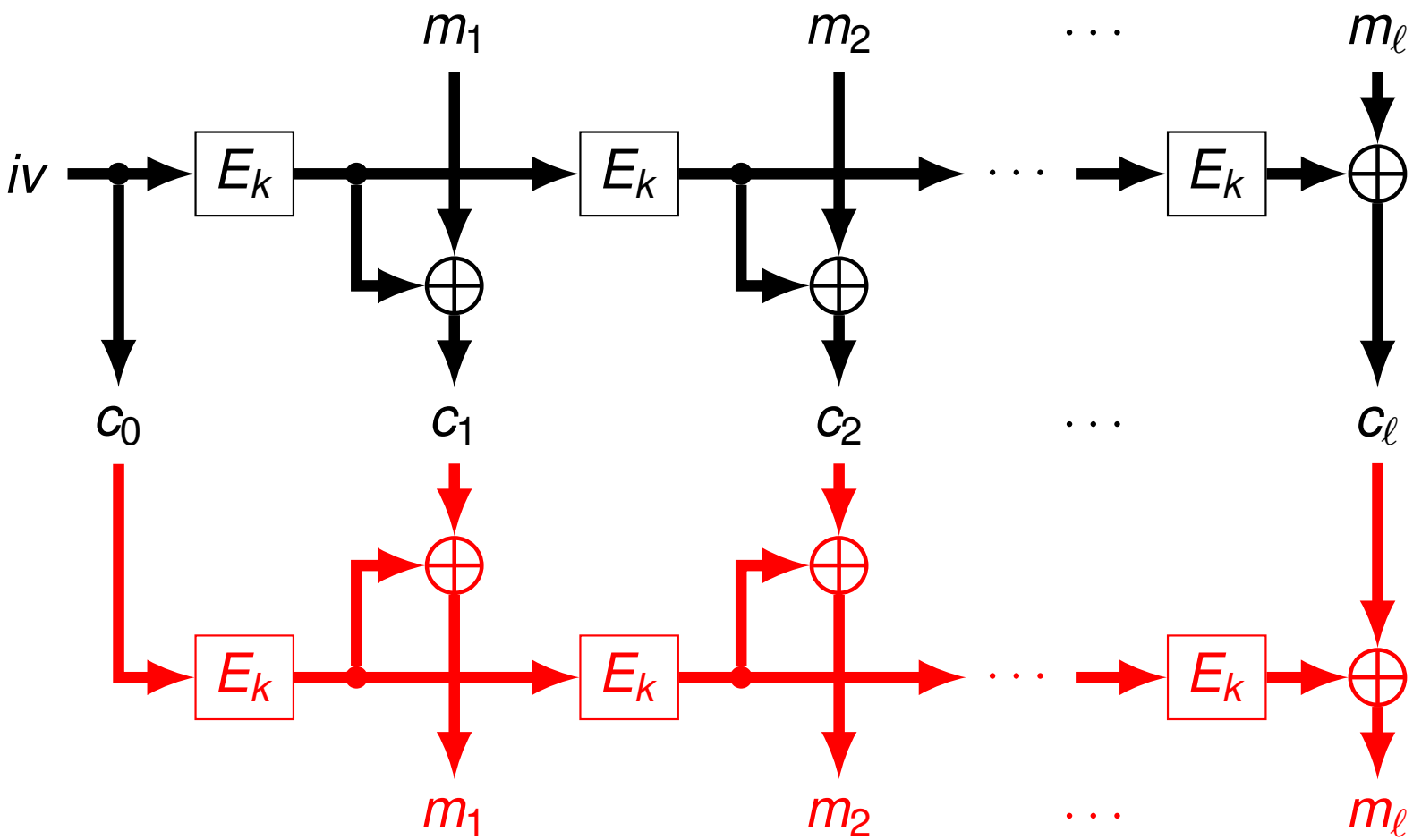
OFB Mode

Comp...



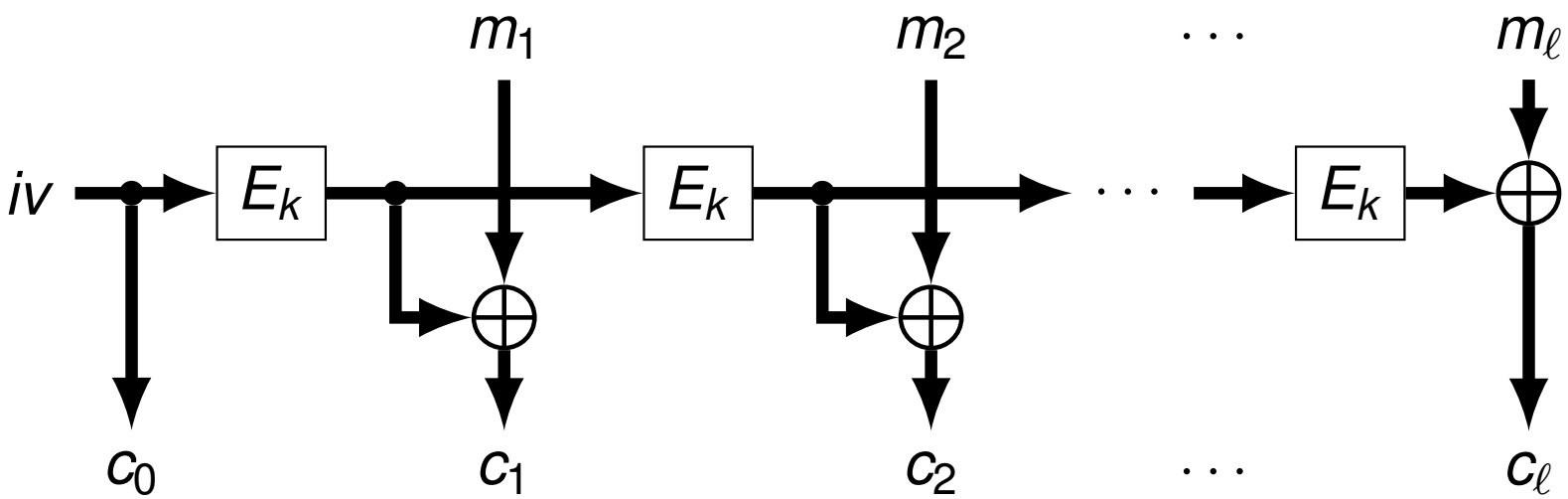
OFB Mode

Comp...



OFB Mode

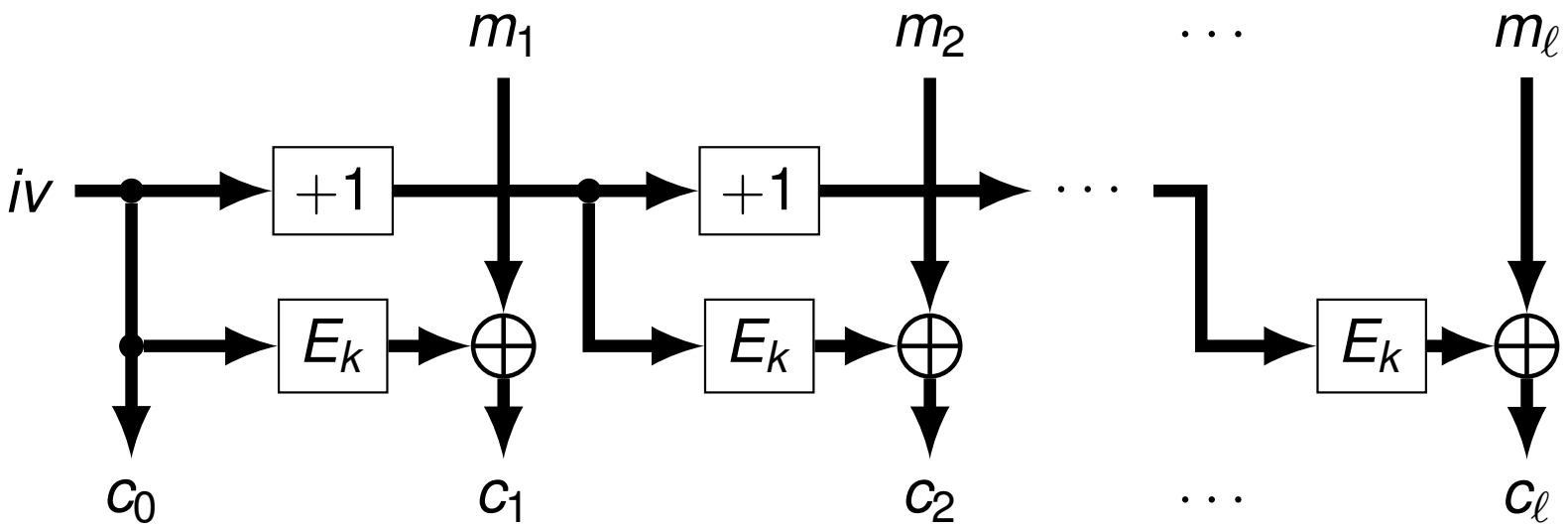
Comp...



A short cycle of E_k of length r reveals $m_i \oplus m_{i+r}$ for all i

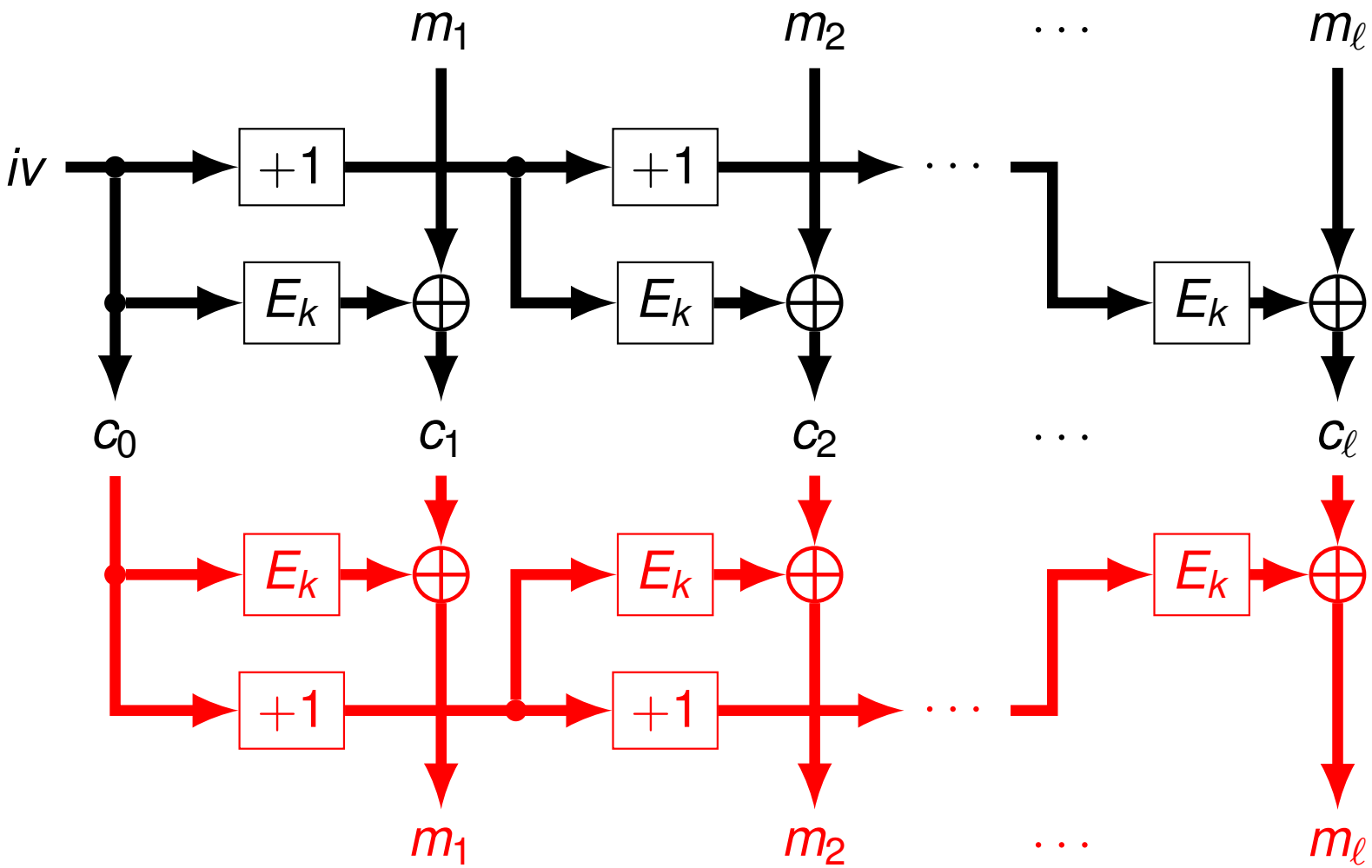
CTR Mode

Comp. . .



CTR Mode

Comp...



Comparison of Block Chaining Modes

- CFB, OFB and CTR require only the encryption box (even for decryption)
- CBC and CFB are “self-synchronizable”
- CTR can use random access for encryption and decryption (i.e., it is parallelizable)
- CBC and CFB decryption are also parallelizable (random access)
- A corrupted *iv* in OFB and CTR makes all blocks undecryptable
- Only ECB and CBC require padding of the last incomplete message block

CBC... CFB... OFB... CTR...

Plaintext Padding

If the last message block is not complete, a bit string is appended such that:

- It has the minimum possible length.
- It has to not introduce any ambiguity in decryption.

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Example:

- Append a string: “1” followed by zero or more “0”.
- The empty string is not a valid padding. A last complete block also needs to be padded!
- In decryption, the last “1” and all trailing zeros (if any) are discarded.

Outline

1 Stream Ciphers

2 Block Ciphers

3 Block Chaining Modes

4 Key Management

5 Message Authentication Codes

Key Generation and Storage Issues

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Keys have to be stored securely (Then, new keys required)

- Use hierarchical encryption: encrypt some keys under a common dedicated key.
- Can incur in **circularity**: plaintext depending on the key!
- Use a **Key Derivation Function** applied to a passphrase and a (public) nonce.

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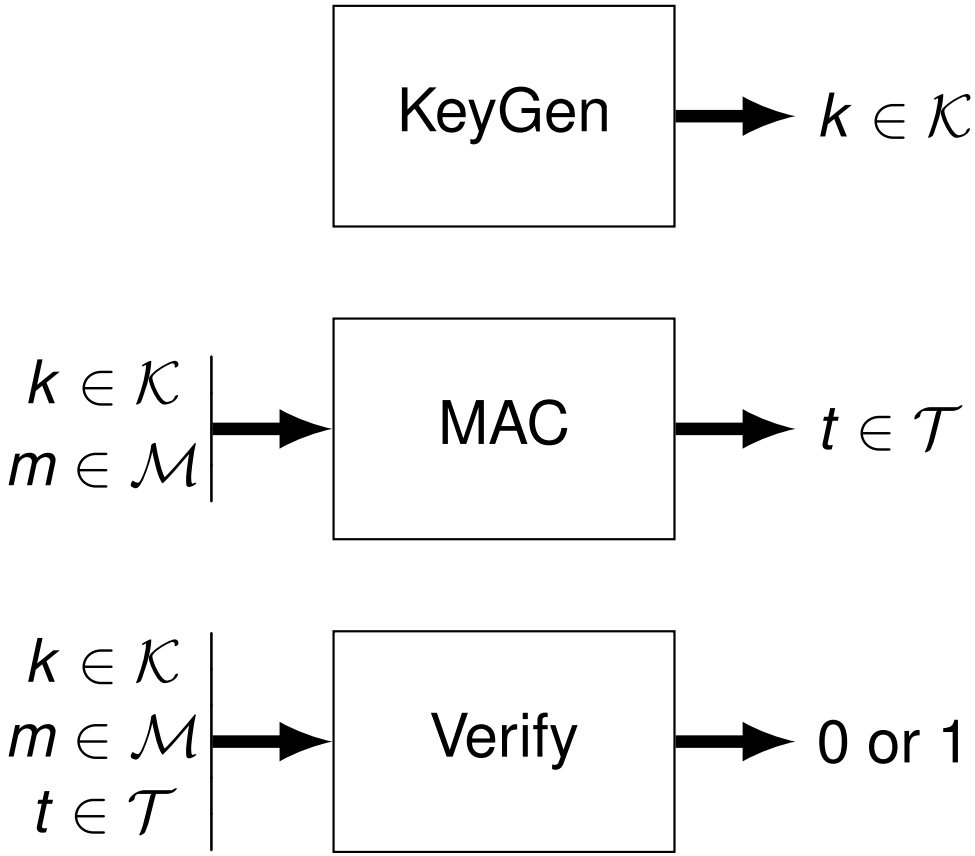
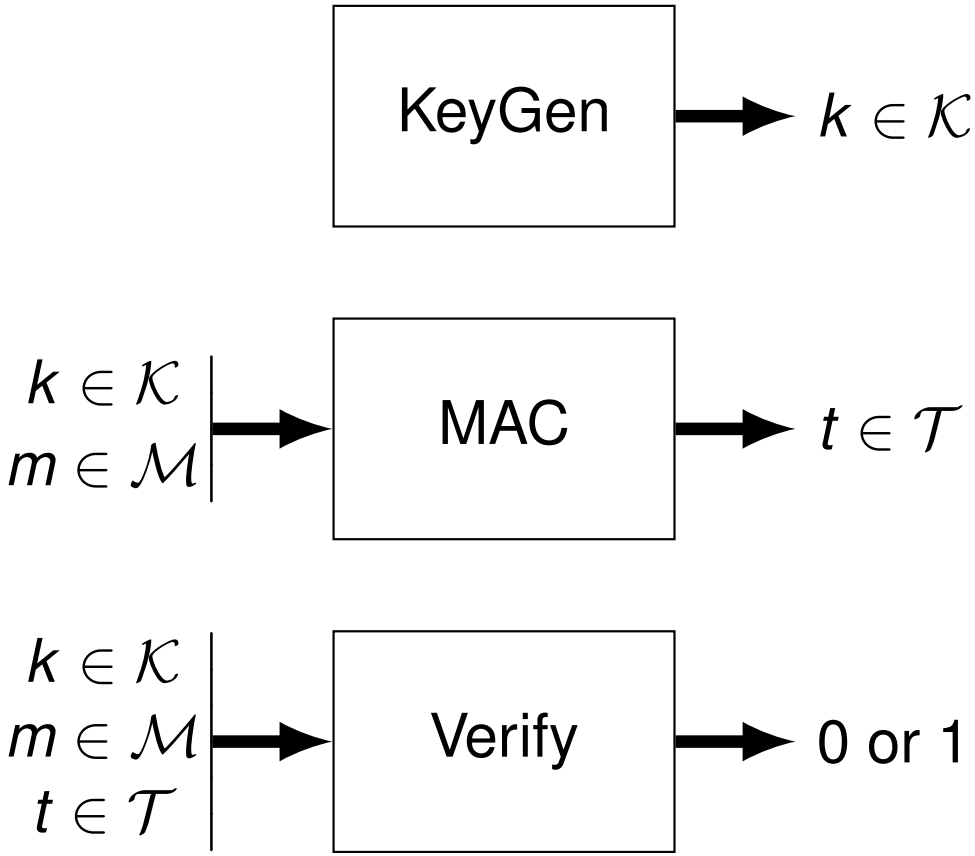
Message Authentication Codes: Syntax

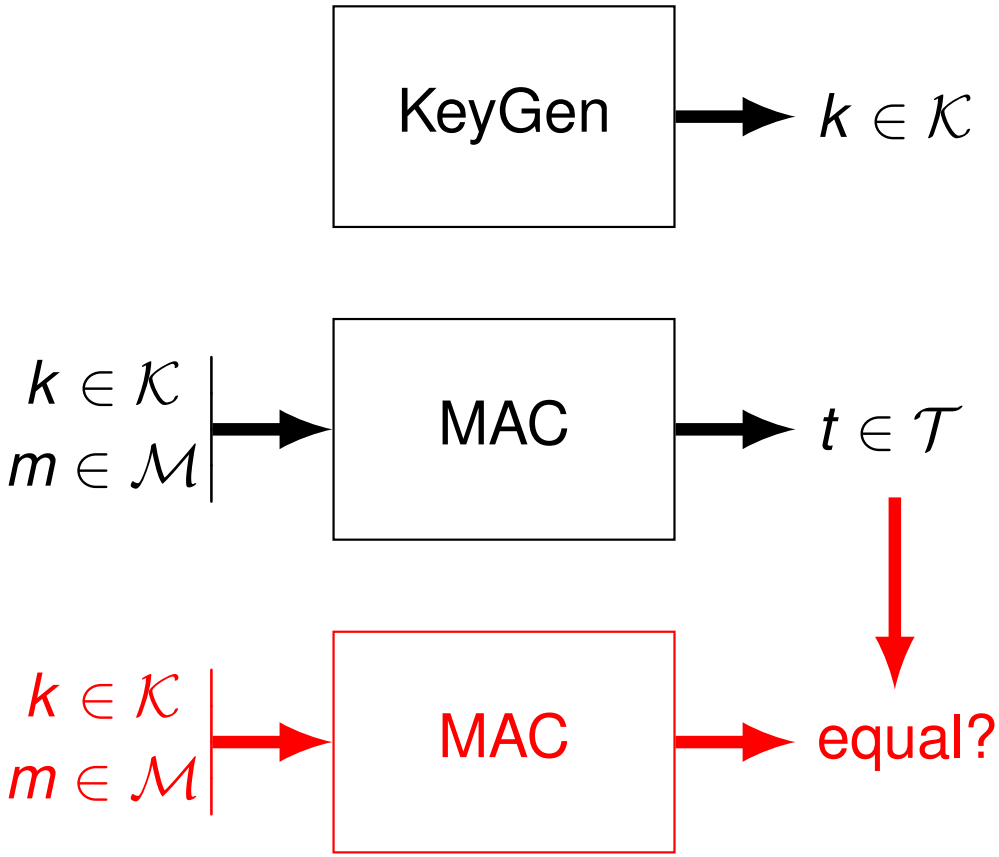


Message Authentication Codes: Correctness



$$\forall m \in \mathcal{M},\ \forall k \in \mathcal{K}, \quad \text{Verify}(k, m, \text{MAC}(k, m)) = 1$$

Typical Verification Procedure



$$\text{Verify}(k, m, t) = 1 \Leftrightarrow t = \text{MAC}(k, m)$$

MAC: Perfect Unforgeability

Informal definition:

"Impossible to find a new pair (m', t') from (m, t) without k"

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Definition (Perfect One-Time Unforgeability)

For a uniformly distributed $K \in \mathcal{K}$ and for any different $m, m' \in \mathcal{M}$, **the tags** $\mathrm{MAC}(K, m)$ **and** $\mathrm{MAC}(K, m')$ **are independent random variables.**

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Definition (Perfect n-Times Unforgeability)

For a uniformly distributed $K \in \mathcal{K}$ and for any different $m_1, m_2, \ldots, m_n, m' \in \mathcal{M}$, **the** $n+1$ **tags** $T_1 = \mathrm{MAC}(K, m_1), \ldots, T_n = \mathrm{MAC}(K, m_n)$ **and** $T' = \mathrm{MAC}(K, m')$ **are independent random variables.**

One-Time Perfect MAC

q is a ℓ-bit long prime, $\mathcal{M} = \mathcal{T} = \mathbb{Z}_q = \{0, 1, \ldots, q-1\}$ and $\mathcal{K} = \mathbb{Z}_q \times \mathbb{Z}_q$.

Construction

Public Parameters(ℓ) :
Choose a ℓ-bit long prime q;
output $pp = (q)$;

KeyGen(pp) :
Parse $pp = (q)$;
Choose random $a, b \leftarrow \mathbb{Z}_q$;
output $k = (a, b)$;

MAC(k, m) :
Parse $k = (a, b)$;
output $t = am + b \mod q$;

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Unforgeability (one time): Even knowing a valid pair $(m, t = am + b)$, still $t' = am' + b$ remains random for any $m' \neq m$.

n-Times Perfect MAC

Now $\mathcal{K} = \mathbb{Z}_q^{n+1} = \underbrace{\mathbb{Z}_q \times \cdots \times \mathbb{Z}_q}_{n+1 \text{ copies}}$.

Construction

Public Parameters(ℓ) : (as before)

KeyGen(pp) :
Choose random $a_1, \ldots, a_n, b \leftarrow \mathbb{Z}_q$;
output $k = (a_1, \ldots, a_n, b)$;

MAC(k, m) :
output $t = P(m) = a_n m^n + \ldots + a_1 m + b \mod q$;

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Unforgeability (n times): Even knowing n valid pairs $(m_1, t_1 = P(m_1)), \ldots, (m_n, t_n = P(m_n))$, still $t' = P(m')$ remains random for any $m' \notin \{m_1, \ldots, m_n\}$. (By Lagrange polynomial interpolation.)

Practical MAC Constructions

Perfect MACs are not practical:

- The key is larger than the message.
- Even in the n-times secure MAC, the key is larger than the size of the n messages together.
- The tag has the same size as the message.

Known efficient (non-perfect) constructions are based on

- hash functions (see next topic)
- block ciphers
- ...

Block Cipher Based MAC

CBC-MAC is a MAC based on any block cipher in CBC mode.

- A short long-term key is used to compute many tags.
- The tag length is independent of the message length.
- The computational complexity is similar to encrypting the message.

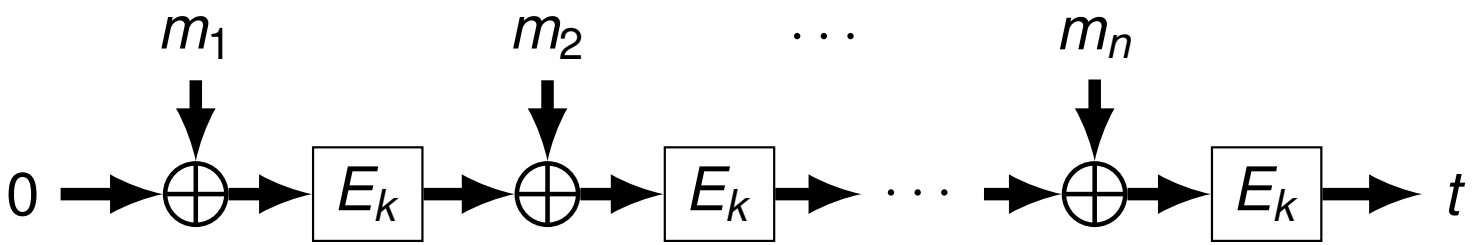
Construction

Public Parameters(ℓ) :
Choose a block cipher with message block size ℓ and a padding mechanism;

KeyGen(pp) :
Generate a random key k for the block cipher;
output k;

MAC(k, m) :
Encrypt (padded) m with key k using CBC mode and taking $iv = 0$;
output the last block of the encryption;

CBC-MAC Diagram



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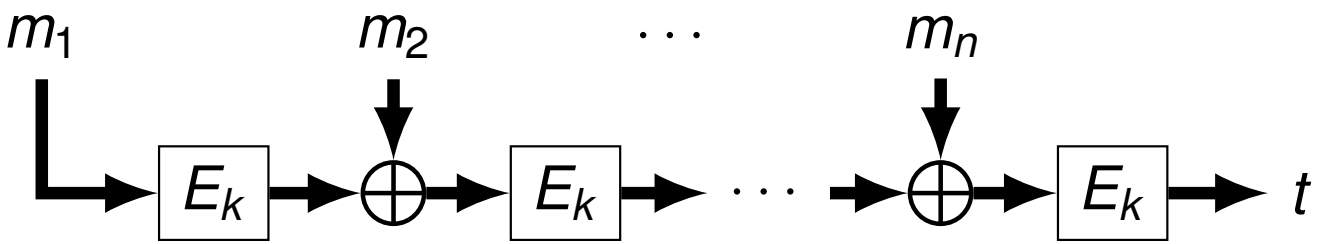

Unforgeability:

- CBC-MAC for messages of a fixed length (exact multiple of the block size) is **proven** to be as secure as the block cipher in CBC mode.
  Any attack against CBC-MAC implies a similar attack against the encryption scheme.

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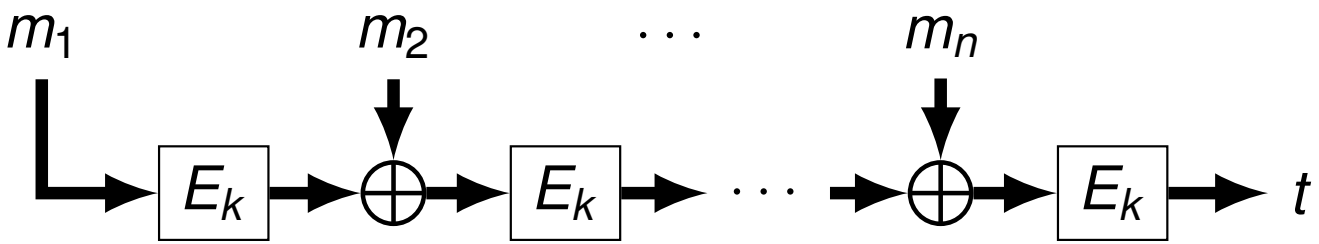
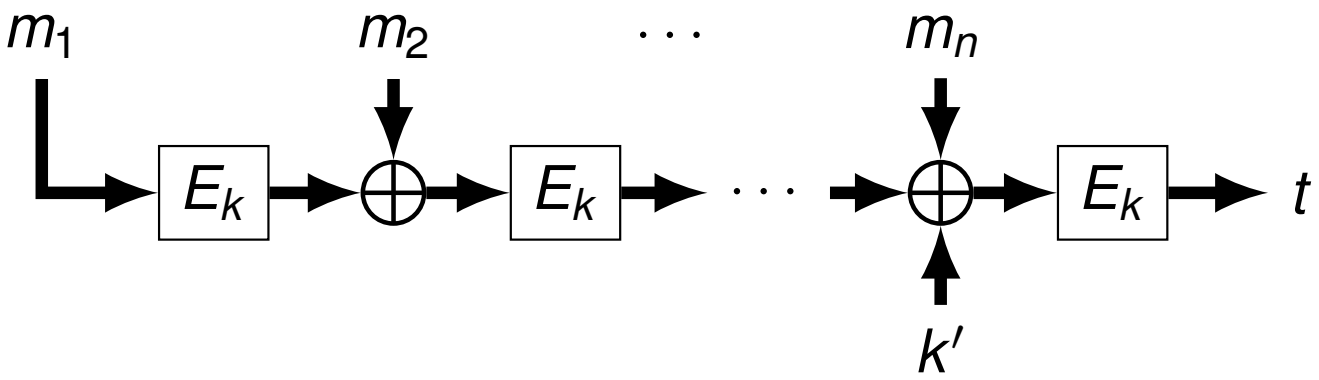


Unforgeability:

- CBC-MAC for messages of a fixed length (exact multiple of the block size) is **proven** to be as secure as the block cipher in CBC mode.
  Any attack against CBC-MAC implies a similar attack against the encryption scheme.
- A known (concatenation) attack exists against CBC-MAC for any block cipher.
  $t = \text{MAC}(k, m_1, \ldots, m_n) = \text{MAC}(k, m_1, \ldots, m_n, m_1 \oplus t, m_2 \ldots, m_n)$

One-Key MAC



CMAC solves the variable length security problem of CBC-MAC without increasing the size of the key.

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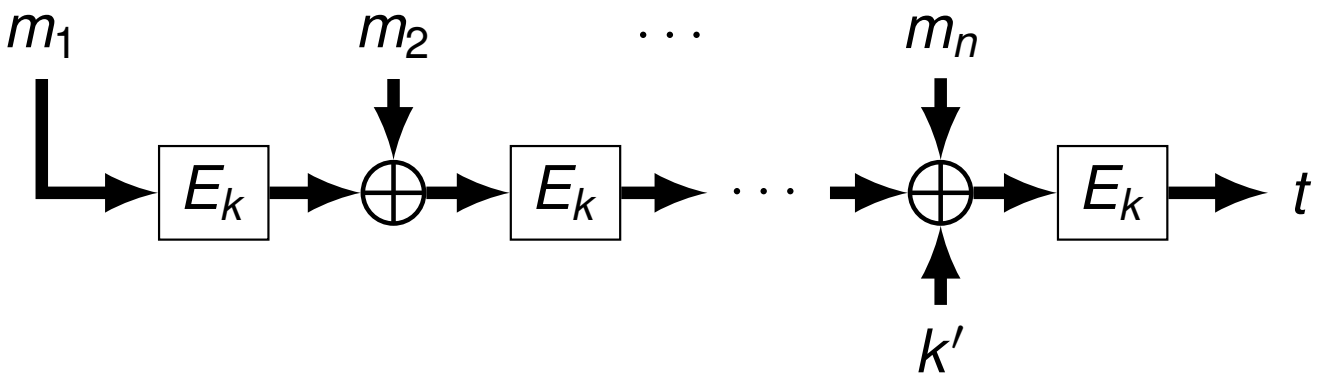

CMAC solves the variable length security problem of CBC-MAC without increasing the size of the key.

- The last message block is “tweaked” with a key k' derived from k.
- Two different values of k' are generated: one is used when the last block requires to be padded, and the other when no padding is necessary.

Cryptology

Jorge L. Villar

FME, UPC, Fall 2024

END