9) Determinem el Triedre de Frenet en $t=1$ de la corba

$\gamma(t) = (1+t, -t^2, 1+\frac{2}{3}t^3)$.

Hem de calcular $\{\vec{T}(1), \vec{N}(1), \vec{B}(1)\}$ (vectors tangent unitari, normal i binormal en $t=1$).

$\dot{\gamma}(t) = (1, -2t, 2t^2)$, $\dot{\gamma}(1) = (1, -2, 2)$, $\|\dot{\gamma}(1)\| = 3$

$\ddot{\gamma}(t) = (0, -2, 4t)$, $\ddot{\gamma}(1) = (0, -2, 4)$

$\dddot{\gamma}(t) = (0, 0, 4)$, $\dddot{\gamma}(1) = (0, 0, 4)$

Així:

$$\vec{T}(1) = \frac{\dot{\gamma}(1)}{\|\dot{\gamma}(1)\|} = \frac{(1,-2,2)}{3}$$

$$\dot{\gamma}(1) \wedge \ddot{\gamma}(1) = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 0 & -2 & 4 \end{vmatrix} = (-4,-4,-2) = 2(-2,-2,-1)$$

$$\|\dot{\gamma}(1) \wedge \ddot{\gamma}(1)\| = 6$$

$$\vec{B}(1) = \frac{\dot{\gamma}(1) \wedge \ddot{\gamma}(1)}{\|\dot{\gamma}(1) \wedge \ddot{\gamma}(1)\|} = \frac{(-2,-2,-1)}{3}$$

$$\vec{N}(1) = \vec{B}(1) \wedge \vec{T}(1) = \begin{vmatrix} i & j & k \\ -2/3 & -2/3 & -1/3 \\ 1/3 & -2/3 & 2/3 \end{vmatrix} = \frac{-1}{9} \begin{vmatrix} i & j & k \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{vmatrix} = -\frac{1}{9}(6,-3,-6) =$$

$$= \frac{(-2,1,2)}{3}$$