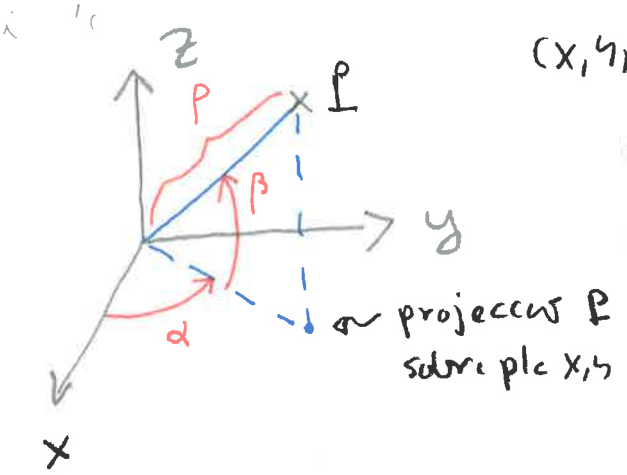
(26) a) Passen de coord. rectangulars a esfèriques els següents punts de l'esfera de radi unitat

$P_1=(-1,0,0)$, $P_2=\left(-\frac{\sqrt{3}}{2\sqrt{2}},\frac{\sqrt{3}}{2\sqrt{2}},\frac{1}{2}\right)$, $P_3=\left(-\frac{1}{2\sqrt{2}},-\frac{\sqrt{3}}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$




$(x,y,z)\in\mathbb{R}^2 \rightsquigarrow (\rho,\alpha,\beta)$ coord. esfèriques

$$\begin{cases} x=\rho\cos\beta\cos\alpha \\ y=\rho\cos\beta\sin\alpha \\ z=\rho\sin\beta \end{cases}$$

$\rho=\sqrt{x^2+y^2+z^2}\in[0,+\infty]$ distància origen

$\sin\beta=\frac{z}{\rho}$, $\beta\in[-\pi/2,\pi/2]$ latitud, $\tan\alpha=\frac{y}{x}$, $\alpha\in[0,2\pi]$ longitud

$\boxed{P_1}$ $\rho=\sqrt{1^2+0^2+0^2}=1$; $\sin\beta=\frac{0}{1}\Rightarrow\beta=0$; $\tan\alpha=\frac{0}{-1}\Rightarrow\alpha=\pi$

$\boxed{P_2}$ $\rho=\sqrt{\left(\frac{\sqrt{3}}{2\sqrt{2}}\right)^2+\left(\frac{\sqrt{3}}{2\sqrt{2}}\right)^2+\left(\frac{1}{2}\right)^2}=1$; $\sin\beta=\frac{1/2}{1}=\frac{1}{2}\Rightarrow\beta=\pi/6$

$\tan\alpha=\frac{\sqrt{3}/2\sqrt{2}}{-\sqrt{3}/2\sqrt{2}}=\frac{1}{-1}=-1\Rightarrow\alpha=\frac{\pi}{4}+\pi=\frac{3\pi}{4}$ (ja que $\cos\alpha<0$, $\sin\alpha>0$)

$\boxed{P_3}$ $\rho=\sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2+\left(\frac{\sqrt{3}}{2\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}=1$; $\sin\beta=\frac{1/\sqrt{2}}{1}=\frac{\sqrt{2}}{2}\Rightarrow\beta=\pi/4$

$\tan\alpha=\frac{-\sqrt{3}/2\sqrt{2}}{-1/2\sqrt{2}}=\frac{-\sqrt{3}}{-1}=\frac{-\sqrt{3}/2}{-1/2}=\sqrt{3}\Rightarrow\alpha=\frac{\pi}{3}+\pi=\frac{4\pi}{3}$ (ja que $\sin\alpha<0$, $\cos\alpha<0$)

(ii) Passen de coord. esfèriques a rectangulars els punts de l'esfera unitat $Q_1=(4\pi/3,-\pi/4)$, $Q_2=(\pi,\pi/8)$, $Q_3=(3\pi/2,\pi/6)$

$\boxed{Q_1}$ $(x,y,z)=\left(1\cdot\cos(-\pi/4)\cos(4\pi/3),\ 1\cdot\cos(-\pi/4)\sin\left(\frac{4\pi}{3}\right),\ 1\cdot\sin(-\pi/4)\right)=$
$=\left(-\sqrt{2}/4,-\sqrt{6}/4,-\sqrt{2}/2\right)$ on usem $\cos\left(\frac{4\pi}{3}\right)=\cos\left(\frac{\pi}{3}+\pi\right)=-\cos\left(\frac{\pi}{3}\right)$; $\sin\left(\frac{4\pi}{3}\right)=-\sin\left(\frac{\pi}{3}\right)$

$\boxed{Q_2}$ $(x,y,z)=\left(1\cdot\cos\left(\frac{\pi}{8}\right)\cdot\cos(\pi),\ 1\cdot\cos\left(\frac{\pi}{8}\right)\cdot\sin(\pi),\ 1\cdot\sin\left(\frac{\pi}{8}\right)\right)=\left(-\frac{\sqrt{2+\sqrt{2}}}{2},0,\frac{\sqrt{2-\sqrt{2}}}{2}\right)$

on: $\sin^2\left(\frac{\pi}{8}\right)=\frac{1-\cos(\pi/4)}{2}=\frac{1-\sqrt{2}/2}{2}=\frac{2-\sqrt{2}}{4}$, $\cos^2\left(\frac{\pi}{8}\right)=\frac{1+\cos(\pi/4)}{2}=\frac{2+\sqrt{2}}{4}$

$\boxed{Q_3}$ $(x,y,z)=\left(1\cdot\cos(\pi/6)\cos(3\pi/2),\ 1\cdot\cos(\pi/6)\cdot\sin\left(\frac{3\pi}{2}\right),\ 1\cdot\sin\left(\frac{\pi}{6}\right)\right)=\left(0,-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$