(22) trobem base ortonormal dels s.e.v. següents:

(i) $F_1 = \{ x - y + 2z = 0 \}$

- Apliquem ortogonalització Gram-Schmidt (en tots els apartats)
- $F_1 = [\vec{u}_1, \vec{u}_2]$, $\vec{u}_1 = (1,1,0)$, $\vec{u}_2 = (-2,0,1)$ solucions eq.
- $F_1 = [\vec{v}_1, \vec{v}_2)$ base ortogonal on $\vec{v}_1 = \vec{u}_1$ i

$\vec{v}_2 = \lambda_1 \vec{v}_1 + \vec{u}_2 = 1 \cdot (1,1,0) + (-2,0,1) = (-1,1,1)$, on

$$\lambda_1 = -\frac{\langle \vec{u}_2, \vec{v}_1 \rangle}{\langle \vec{v}_1, \vec{v}_1 \rangle} = -\frac{(-2)}{2} = 1$$

- $F_1 = [\vec{w}_1, \vec{w}_2]$ b.o.n. on $\vec{w}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{(1,1,0)}{\sqrt{2}}$, $\vec{w}_2 = \frac{\vec{v}_2}{\|\vec{v}_2\|} = \frac{(-1,1,1)}{\sqrt{3}}$

(ii) $F_2 = [(1,1,1)]^{\perp}$

- Eq. de F_2 és $x + y + z = 0 \Rightarrow F_2 = [\vec{u}_1, \vec{u}_2]$ on

$\vec{u}_1 = (-1,1,0)$, $\vec{u}_2 = (-1,0,1)$ solucions eq.

- $F_1 = [\vec{v}_1, \vec{v}_2]$ base ortogonal on $\vec{v}_1 = \vec{u}_1$ i

$\vec{v}_2 = \lambda_1 \vec{v}_1 + \vec{u}_2 = -\frac{1}{2}(-1,1,0) + (-1,0,1) = \frac{1}{2}(-1,-1,2)$ on

$$\lambda_1 = -\frac{\langle \vec{u}_2, \vec{v}_1 \rangle}{\langle \vec{v}_1, \vec{v}_1 \rangle} = -\frac{1}{2}$$

- $F_2 = [\vec{w}_1, \vec{w}_2)$ b.o.n. on $\vec{w}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{(-1,1,0)}{\sqrt{2}}$, $\vec{w}_2 = \frac{\vec{v}_2}{\|\vec{v}_2\|} = \frac{(-1,-1,2)}{\sqrt{6}}$

(iii) $F_3 = [\underbrace{(1,1,2)}_{= \vec{u}_1}, \underbrace{(2,3,-1)}_{= \vec{u}_2}]$

- $F_3 = [\vec{v}_1, \vec{v}_2]$ base ortogonal on $\vec{v}_1 = \vec{u}_1$ i

$\vec{v}_2 = \lambda_1 \vec{v}_1 + \vec{u}_2 = -\frac{1}{2}(1,1,2) + (2,3,-1) = (3/2, 5/2, -2) = \frac{1}{2}(3,5,-4)$ on

$$\lambda_1 = -\frac{\langle \vec{u}_2, \vec{v}_1 \rangle}{\langle \vec{v}_1, \vec{v}_1 \rangle} = -\frac{3}{6} = -\frac{1}{2}$$

- $F_3 = [\vec{w}_1, \vec{w}_2]$ b.o.n. on $\vec{w}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{(1,1,2)}{\sqrt{6}}$, $\vec{w}_2 = \frac{\vec{v}_2}{\|\vec{v}_2\|} = \frac{(3,5,-4)}{5\sqrt{2}}$

(iv) $F_4 = [\underbrace{(2,1,2)}_{\vec{u}_1}, \underbrace{(0,3,1)}_{\vec{u}_2}]$

· $F_4 = [\vec{v}_1, \vec{v}_2]$ base ortogonal on $\vec{v}_1 = \vec{u}_1$ i

$\vec{v}_2 = \lambda_1 \vec{v}_1 + \vec{u}_2 = -\frac{5}{9}(2,1,2) + (0,3,1) = \frac{1}{9}(-10,22,-1)$ i

$\lambda_1 = -\frac{\langle \vec{u}_2, \vec{v}_1 \rangle}{\langle \vec{v}_1, \vec{v}_1 \rangle} = -\frac{5}{9}$

· $F_4 = [\vec{w}_1, \vec{w}_2]$ b.o.n. on $\vec{w}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{(2,1,2)}{3}$, $\vec{w}_2 = \frac{\vec{v}_2}{\|\vec{v}_2\|} = \frac{(-10,22,-1)}{3\sqrt{65}}$

(v) $F_5 = \begin{cases} x+y-z-t=0 \\ x-y-z+t=0 \end{cases} \Leftrightarrow \begin{cases} 2x-2z=0 \\ 2y-2t=0 \end{cases} \Leftrightarrow \begin{cases} x=z \\ y=t \end{cases}$

sumem/restem eqns.

· $F_5 = [\vec{u}_1, \vec{u}_2]$ on $\vec{u}_1 = (1,0,1,0)$, $\vec{u}_2 = (0,1,0,1)$

ja són ortogonals!

· $F_5 = [\vec{w}_1, \vec{w}_2]$ b.o.n on $\vec{w}_1 = \frac{\vec{u}_1}{\|\vec{u}_1\|} = \frac{(1,0,1,0)}{\sqrt{2}}$, $\vec{w}_2 = \frac{\vec{u}_2}{\|\vec{u}_2\|} = \frac{(0,1,0,1)}{\sqrt{2}}$