(5) Siguin $\Pi_1 = \{2x - y + z - 10 = 0\}$ i $\Pi_2 = \{x + z + 4 = 0\}$ dos plans de $\mathbb{R}^3$ i sigui $P = (0,1,-1)$ un punt.

(i) Trobeu P_1, P_2 projeccions ortogonals de P sobre Π_1 i Π_2, respectivament.

- $P_1 = \Pi_{\Pi_1}(P) = a_1 + \Pi_{F_1}(\overrightarrow{a_1P}) = a_1 + [\overrightarrow{a_1P} - \Pi_{F_1^\perp}(\overrightarrow{a_1P})] =$

$$= a_1 + P - a_1 - \Pi_{F_1^\perp}(\overrightarrow{a_1P}) = P - \Pi_{F_1^\perp}(\overrightarrow{a_1P})$$

on $a_1 \in \Pi_1$ (punt qualsevol), $F_1 = [2x - y + z = 0]$ s.e.v. director de Π_1 i $F_1^\perp = [(2,-1,1)]$. Per tant:

Fent $y = z = 0 \Rightarrow 2x = 10 \Rightarrow x = 5 \Rightarrow a_1 = (5,0,0)$.

$\overrightarrow{a_1P} = P - a_1 = (0,1,-1) - (5,0,0) = (-5,1,-1)$

$$\Pi_{F_1^\perp}(\overrightarrow{a_1P}) = \Pi_{[(2,-1,1)]}(-5,1,-1) = \frac{\langle(-5,1,-1),(2,-1,1)\rangle}{\langle(2,-1,1),(2,-1,1)\rangle}(2,-1,1) =$$

$$= \frac{-12}{6}(2,-1,1) = (-4,2,-2)$$

$P_1 = P - \Pi_{F_1^\perp}(\overrightarrow{a_1P}) = (0,1,-1) - (-4,2,-2) = (4,-1,1)$

- $P_2 = \Pi_{\Pi_2}(P) = P - \Pi_{F_2^\perp}(\overrightarrow{a_2P})$, on:

$a_2 \in \Pi_2$, $F_2 = [x + z = 0]$, $F_2^\perp = [(1,0,1)]$.

Fem $y = z = 0 \Rightarrow x = -4 \Rightarrow a_2 = (-4,0,0)$.

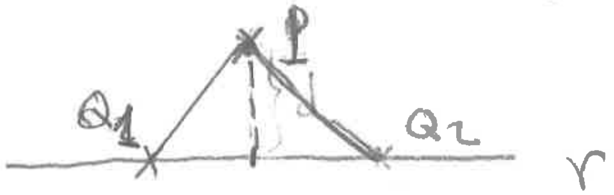
$\overrightarrow{a_2P} = P - a_2 = (0,1,-1) - (-4,0,0) = (4,1,-1)$

$$\Pi_{F_2^\perp}(\overrightarrow{a_2P}) = \Pi_{[(1,0,1)]}(4,1,-1) = \frac{\langle(4,1,-1),(1,0,1)\rangle}{\langle(1,0,1),(1,0,1)\rangle}(1,0,1) =$$

$$= \frac{3}{2}(1,0,1)$$

$P_2 = P - \Pi_{F_2^\perp}(\overrightarrow{a_2P}) = (0,1,-1) - \frac{3}{2}(1,0,1) = (-3/2, 1, -5/2)$

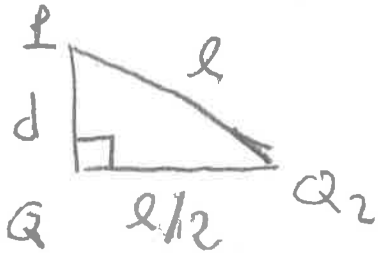
(ii) Trobem punts Q_1, Q_2 sobre la recta $r = \Pi_1 \cap \Pi_2$ t.q. el triangle P, Q_1, Q_2 sigui equilater.



$Q = \Pi_r(P)$

· Q és el punt mig entre Q_1 i Q_2.

· Si $d = d(P, Q)$ llavors



Això és, l longitud costat triangle $\Rightarrow$ $l = d(P, Q_2)$
$l/2 = d(Q, Q_2)$

$$d^2 + (l/2)^2 = l^2 \Rightarrow d^2 = l^2 - l^2/4 = \frac{3}{4} l^2 \Rightarrow l = \frac{2}{\sqrt{3}} d.$$

Per tant, si v és un vector unitari director de r, tenim: $Q_1 = Q + \frac{l}{2} v$, $Q_2 = Q - \frac{l}{2} v$

· Busquem un vector director de r:

$$\left.\begin{array}{l}(2,-1,1) \perp \Pi_1 \\ (1,0,1) \perp \Pi_2\end{array}\right\} (2,-1,1) \wedge (1,0,1) = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = (-1,-1,1) \parallel r$$

Triem $(-1,-1,1)$ director $r \Rightarrow \underset{\substack{\uparrow \\ \text{director unitari}}}{v} = \frac{(-1,-1,1)}{\|(-1,-1,1)\|} = \frac{(-1,-1,1)}{\sqrt{3}}$

· Busquem $a \in r$ solució de:

$$\left.\begin{array}{l}2x - y + z - 10 = 0 \\ x + z + 4 = 0\end{array}\right\} \text{Fent, p. ex. } x = 0 \Rightarrow z = -4,\ y = -14$$
$a = (0,-14,-4)$

· Busquem $Q = \Pi_r(P) = a + \Pi_F(\overrightarrow{aP})$, on $F = [(-1,-1,1)]$ s.e.v. director de r.

$\overrightarrow{aP} = P - a = (0,1,-1) - (0,-14,-4) = (0,15,3)$

$$\Pi_F(\vec{aP}) = \Pi_{[(-1,-1,1)]}(0,15,3) = \frac{\langle (0,15,3), (-1,-1,1) \rangle}{\langle (-1,-1,1), (-1,-1,1) \rangle}(-1,-1,1) =$$

$$= \frac{-12}{3}(-1,-1,1) = (4,4,-4)$$

$$Q = \Pi_r(P) = a + \Pi_F(\vec{aP}) = (0,-14,-4) + (4,4,-4) = (4,-10,-8)$$

$$d = d(P,Q) = \| Q - P \| = \| (4,-11,-7) \| = \sqrt{4^2+11^2+7^2} = \sqrt{186}$$

$$l = \frac{2}{\sqrt{3}} d = \frac{2\sqrt{186}}{\sqrt{3}} = 2\sqrt{62}$$

Finalment:

$$Q_1 = Q + \frac{l}{2} v = (4,-10,-8) + \frac{2\sqrt{62}}{2} \frac{(-1,-1,1)}{\sqrt{3}} = (4,-10,-8) + \frac{\sqrt{62}}{\sqrt{3}}(-1,-1,1)$$

$$Q_2 = Q - \frac{l}{2} v = (4,-10,-8) - \frac{\sqrt{62}}{\sqrt{3}}(-1,-1,1)$$

Podem verificar $d(Q_1, Q_2) = d(Q_1, P) = d(Q_2, P) = 2\sqrt{62}$.