

① Calcular el s.e.v. ortogonal als següents s.e.v.:

(i) $F_1 = [(2, -1, 5), (1, 2, 8)] \subset \mathbb{R}^3$

Eqms: $F_1^\perp = \{2x - y + 5z = 0, x + 2y + 8z = 0\}$ (els coeficients són els dos vectors de F_1)

base: $\dim F_1^\perp = 1$ ve generat per $F_1^\perp = [(-18, -11, 5)]$

$(2, -1, 5) \wedge (1, 2, 8) = \begin{vmatrix} i & j & k \\ 2 & -1 & 5 \\ 1 & 2 & 8 \end{vmatrix} = \begin{vmatrix} -1 & 5 \\ 2 & 8 \end{vmatrix} i - \begin{vmatrix} 2 & 5 \\ 1 & 8 \end{vmatrix} j + \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} k = (-18, -11, 5)$

(ii) $F_2 = \{x + 8y - 4z = 0\}$

base: $F_2^\perp = [(1, 2, -4)]$ (via coef. eq F_2)

Eqms: $F_2^\perp = \left\{ \frac{x}{1} = \frac{y}{2} = \frac{z}{-4} \right\} = \{2x - y = 0, 4x + z = 0\}$

obtingudes com recta pel $(0, 0, 0)$ i vector director $(1, 2, -4)$

(iii) $F_3 = \{x_1 + x_2 - x_3 - x_4 = 0, x_1 - 3x_2 + 4x_3 - 2x_4 = 0\} \subset \mathbb{R}^4$

base: $F_3^\perp = [(1, 1, -1, -1), (1, -3, 4, -2)]$ (via coef. eqms F_3)

eqms. (com pla on \mathbb{R}^4 pel $(0, 0, 0, 0)$)

$$\left(\begin{array}{cc|c} 1 & 1 & x_1 \\ 1 & -3 & x_2 \\ -1 & 4 & x_3 \\ -1 & -2 & x_4 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & 1 & x_1 \\ 0 & -4 & x_2 - x_1 \\ 0 & 5 & x_3 + x_1 \\ 0 & -1 & x_4 + x_1 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & 1 & x_1 \\ 0 & 0 & +5x_1 + x_2 - 4x_4 \\ 0 & 0 & +6x_1 + x_3 + 5x_4 \\ 0 & -1 & x_4 + x_1 \end{array} \right)$$

$f_2 = f_2 - f_1$

$f_2 = f_2 + 4f_4$

$f_3 = f_3 + f_1$

$f_3 = f_3 + 5f_3$

$f_4 = f_4 + f_1$

$F_3^\perp = \{-5x_1 + x_2 - 4x_4 = 0, 6x_1 + x_3 + 5x_4 = 0\}$