- Exercici: Classifiqueu qualitativament (node, focus, ...) el sistema d'edo's lineals en $\mathbb{R}^2$ donat per la matriu

$A = \begin{pmatrix} -\mu & 2\mu-1 \\ 1 & 0 \end{pmatrix}$, on $\mu \in \mathbb{R}$ és un paràmetre

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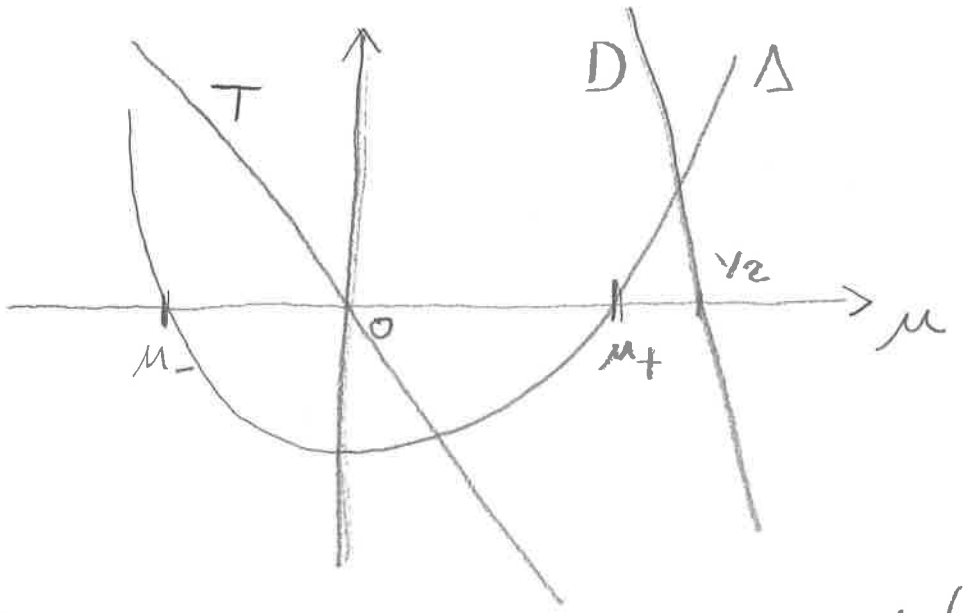
$T = tr(A) = -\mu$; $D = \det(A) = 1-2\mu$

$\Delta = T^2 - 4D = \mu^2 - 4(1-2\mu) = \mu^2 + 8\mu - 4$ (discriminant)

- $T = 0 \Leftrightarrow \mu = 0$; $D = 0 \Leftrightarrow \mu = 1/2$

$\Delta = 0 \Leftrightarrow \mu_\pm = \frac{-8 \pm \sqrt{80}}{2} = -4 \pm 2\sqrt{5}$ ($\mu_- < 0$ i $0 < \mu_+ < \frac{1}{2}$)



- $\boxed{\mu < \mu_-}$ $D > 0$, $\Delta > 0 \Rightarrow$ Node propi ($T > 0 \Rightarrow$ (I))
- $\boxed{\mu_- < \mu < 0}$ $\Delta < 0$, $T \neq 0 \Rightarrow$ focus ($T > 0 \Rightarrow$ (I))
- $\boxed{0 < \mu < \mu_+}$ $\Delta < 0$, $T \neq 0 \Rightarrow$ focus ($\begin{matrix} D>0 \\ T<0 \end{matrix} \Rightarrow$ (AE))
- $\boxed{\mu_+ < \mu < 1/2}$ $D > 0$, $\Delta > 0 \Rightarrow$ Node propi ($\begin{matrix} D>0 \\ T<0 \end{matrix} \Rightarrow$ (AE))
- $\boxed{1/2 < \mu}$ $D < 0 \Rightarrow$ sella ($D < 0 \Rightarrow$ (I))

Falta discutir els "casos crítics":

$\mu = \mu_-$ $\Delta = 0 \Rightarrow$ 2 vap's reals dobles, però A no és diagonal.

Per tant node impropi ($T > 0 \Rightarrow$ (I))

$\mu = 0$ $T = 0$, $D > 0 \Rightarrow$ centre ((E) però no (AE))

$\mu = \mu_+$ $\Delta = 0 \Rightarrow$ 2 vap's reals dobles, però A no és diagonal

node impropi ($D > 0$, $T < 0 \Rightarrow$ (AE))

$\mu = 1/2$ $D = 0 \Rightarrow$ Hi ha un vap $= 0 \Rightarrow$ cas degenerat

($T < 0 \Rightarrow$ (E) no (AE))

$D = 0$