

Sistema de Van der Pol

$$X'' = a \cdot (1 - X^2) \cdot X' - X$$

• Can $a = 0$: Oscilador harmónico $X'' = -X$

• Can $a \neq 0$: Oscilador harmónico não linear.

• Ver par: Sistema de 1er. ordem equivalente

$$\begin{aligned} \begin{cases} X' = y \\ y' = a \cdot (1 - X^2) y - X \end{cases} &::= f(x, y) \\ &::= g(x, y) \end{aligned}$$

• 2en. par: Pontos fixos Sistema 1er. ordem

$$\begin{aligned} f(x, y) = 0 &\Rightarrow y = 0 \\ g(x, y) = 0 &\Rightarrow x = 0 \end{aligned}$$

$(0, 0)$ é o único
pt. fixo

- 3er pas: Sistema linealitzat en $(0,0)$

$$\begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2axy-1 & a(1-x^2) \end{pmatrix}$$

La matriu del sist. linealitzat en $(0,0)$ és

$$A = \begin{pmatrix} 0 & 1 \\ -1 & a \end{pmatrix}$$

- 4art. pas: Estabilitat de $(0,0)$ en el sistema no lineal
via el sistema linealitzat si $a \neq 0$

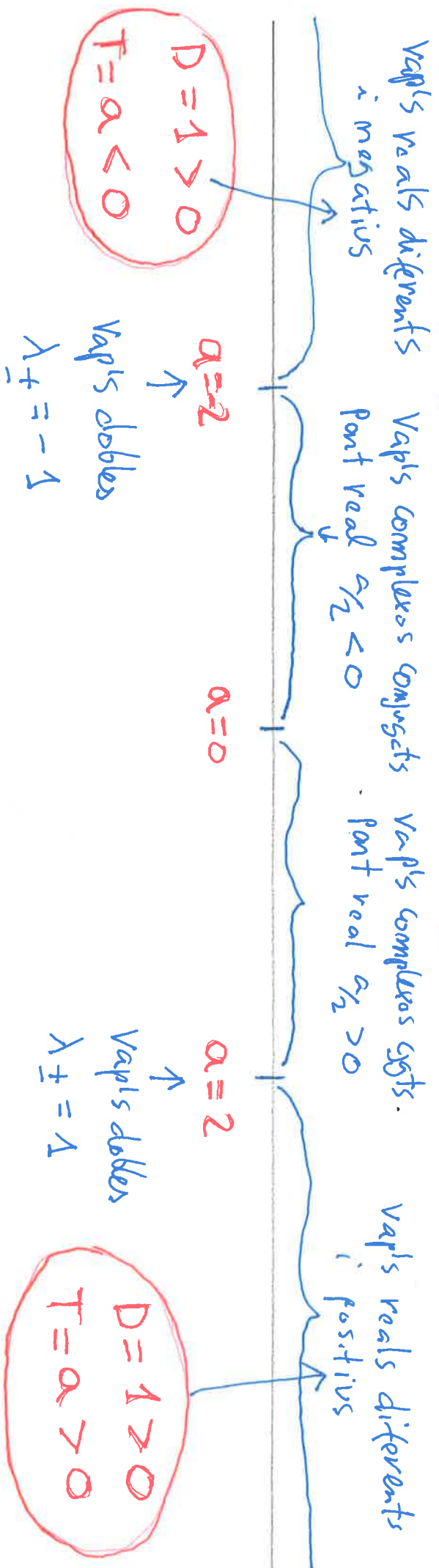
$$T = \text{tr}(A) = a \equiv \text{Suma vap's}$$

$\equiv$ producte vap's

$$D = \det(A) = 1$$

$$P_A(\lambda) = \lambda^2 - T \cdot \lambda + D = \lambda^2 - a \cdot \lambda + 1 \equiv \text{polinomi característic}$$

$$\lambda_{\pm} = \frac{a \pm \sqrt{a^2 - 4}}{2} \equiv \text{vap's de } A$$



- Can $a < -2 \Rightarrow$ El sist. linealitzat té tots els vap's $\ll 0 \Rightarrow$ AE
- Can $a = -2 \Rightarrow$ " " " " " " $< 0 \Rightarrow$ AE
- Can $a \in (-2, 0) \Rightarrow$ " " " " " " amb $\text{Re}(\lambda) < 0 \Rightarrow$ AE
- Can $a \in (0, 2) \Rightarrow$ " " " " " " tots " " " " $> 0 \Rightarrow$ R
- Can $a = -2 \Rightarrow$ " " " " " " tots " " " " $> 0 \Rightarrow$ R
- Can $a > 2 \Rightarrow$ " " " " " " tots " " " " $> 0 \Rightarrow$ R