- Exemple:

$$\bar{X}' = \underbrace{\begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}}_{\substack{\shortparallel \\ A}} \bar{X} + \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{\substack{\shortparallel \\ b(t)}}$$

* Sistema homogeni associat:

$$\bar{X}' = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix} \bar{X}, \quad \underbrace{P_A(\lambda)}_{\text{Polinomi característic}} = \lambda^2 + 1, \quad \text{Vap's}: \lambda_1 = i, \ \lambda_2 = -i$$

$$\text{Vep's}: \bar{v}_1 = (1-i, 1)$$

$$\bar{v}_2 = \overline{\bar{v}_1} = (1+i, 1)$$

- Conjunt fonamental:

$$\begin{cases} \bar{X}_1(t) = e^{0 \cdot t}\left(\cos t \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \sin t \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} \cos t + \sin t \\ \cos t \end{pmatrix} \\ \bar{X}_2(t) = e^{0 \cdot t}\left(\sin t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \cos t \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} \sin t - \cos t \\ \sin t \end{pmatrix} \end{cases}$$

- Matriu fonamental:

$$\Phi(t) = \begin{pmatrix} \cos t + \sin t & \sin t - \cos t \\ \cos t & \sin t \end{pmatrix}, \quad \det \Phi(t) = 1$$

$$(\Phi(t))^{-1} = \begin{pmatrix} \sin t & -\sin t + \cos t \\ -\cos t & \cos t + \sin t \end{pmatrix}$$

* Solució particular edo no homogènia: mètode de variació de les constants.

$$\bar{X}_p(t) = \Phi(t) \int \Phi(t)^{-1} b(t)\, dt = \Phi(t) \int \begin{pmatrix} \sin t \\ -\cos t \end{pmatrix} dt =$$

$$= \Phi(t) \begin{pmatrix} -\cos t + \cancelto{0}{c_1} \\ -\sin t + \cancelto{0}{c_2} \end{pmatrix} = \begin{pmatrix} -\cos^2 t - \cos t \cdot \sin t - \sin^2 t + \sin t \cdot \cos t \\ -\cos^2 t - \sin^2 t \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

* Solució general edo no homogènia:

$$\mathbf{X}(t) = \begin{pmatrix} -1 \\ -1 \end{pmatrix} + C_1 \begin{pmatrix} \cos t + \sin t \\ \cos t \end{pmatrix} + C_2 \begin{pmatrix} \sin t - \cos t \\ \sin t \end{pmatrix}$$

- Exemple de P.V.I.: $\mathbf{X}(\pi) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \mathbf{X}(\pi) = \begin{pmatrix} -1 \\ -1 \end{pmatrix} + C_1 \begin{pmatrix} -1 \\ -1 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{array}{l} 1 = -1 - C_1 + C_2 \\ 1 = -1 - C_1 \end{array}\right\}$$

$C_1 = -2$, $C_2 = 0$

$$\mathbf{X}(t) = \begin{pmatrix} -1 - 2\cos t - 2\sin t \\ -1 - 2\cos t \end{pmatrix}$$