(28) Trobeu el centre de masses de la semi-esfera definida per $x^2+y^2+z^2 \le R^2$ i $z \ge 0$, si la densitat ~~de~~ cada punt és proporcional a la distància d'aquest punt al centre.

$$\rho(x,y,z) = K\sqrt{x^2+y^2+z^2}$$

$$D = \{x^2+y^2+z^2 \le R^2,\ z \ge 0\} \xrightarrow[\text{esfèriques}]{\text{Coord.}} D^* = \left\{ \begin{array}{l} 0 \le r \le R \\ 0 \le \theta \le 2\pi \\ 0 \le \varphi \le \pi/2 \end{array} \right\}$$

- $$m(D) = \iiint_D \rho(x,y,z)\,dx\,dy\,dz = \iiint_{D^*} K \cdot r \cdot \underbrace{r^2 \cos\varphi}_{\text{jacobià}}\, dr\, d\theta\, d\varphi =$$

$$= K\left(\int_0^R r^3 dr\right)\left(\int_0^{2\pi} d\theta\right)\left(\int_0^{\pi/2} \cos\varphi\, d\varphi\right) = K \cdot \frac{R^4}{4} \cdot 2\pi \cdot 1 = \frac{K\pi R^4}{2}$$

- $CM(D) = (\bar{x}, \bar{y}, \bar{z}) = (0, 0, \bar{z}) \leftarrow$ tant D com ρ són simètriques respecte x, y. D no ho és respecte de z.

- $$\bar{z} = \frac{1}{m(D)} \iiint_D z\,\rho(x,y,z)\,dx\,dy\,dz = \frac{K}{m(D)} \iiint_{D^*} r\sin\varphi \cdot r \cdot r^2\cos\varphi\, dr\, d\theta\, d\varphi =$$

$$= \frac{K}{m(D)} \underbrace{\left(\int_0^R r^4 dr\right)}_{= R^5/5} \underbrace{\left(\int_0^{2\pi} d\theta\right)}_{= 2\pi} \underbrace{\left(\int_0^{\pi/2} \sin\varphi \cdot \cos\varphi\, d\varphi\right)}_{= \left[\frac{\sin^2\varphi}{2}\right]_{\varphi=0}^{\varphi=\pi/2} = 1/2} = \frac{K}{K\pi R^4/2} \cdot \frac{R^5}{5} \cdot 2\pi \cdot \frac{1}{2} =$$

$$= \frac{2R}{5}.$$