(13) Calculeu les següents integrals triples iterades.

(a) $I = \int_1^2 \int_0^1 \int_0^{\pi/2} x^2 y^3 \sin z \, dz \, dy \, dx$

$$I = \left(\int_1^2 x^2 dx\right) \cdot \left(\int_0^1 y^3 dy\right) \cdot \left(\int_0^{\pi/2} \sin z \, dz\right) = \left[\frac{x^3}{3}\right]_{x=1}^{x=2} \cdot \left[\frac{y^4}{4}\right]_{y=0}^{y=1} \cdot \left[-\cos z\right]_{z=0}^{\pi/2} =$$

$$= \frac{8-1}{3} \cdot \frac{1-0}{4} \cdot \left((-0)-(-1)\right) = 7/12$$

(b) $I = \int_0^1 \int_0^x \int_0^{\sqrt{x^2+y^2}} z \, dz \, dy \, dx$

$$I = \int_0^1 \int_0^x \left[\frac{z^2}{2}\right]_{z=0}^{z=\sqrt{x^2+y^2}} dy \, dx = \int_0^1 \int_0^x \frac{x^2+y^2}{2} dy \, dx = \int_0^1 \left[\frac{x^2 y}{2} + \frac{y^3}{6}\right]_{y=0}^{y=x} dx = \int_0^1 \left[\frac{x^3}{2} + \frac{x^3}{6}\right] dx =$$

$$= \int_0^1 \frac{2}{3} x^3 dx = \left[\frac{2x^4}{12}\right]_{x=0}^{x=1} = \frac{1}{6}.$$

(c) $I = \int_0^3 \int_0^{2x} \int_0^{\sqrt{xy}} z \, dz \, dy \, dx$

$$I = \int_0^3 \int_0^{2x} \left[\frac{z^2}{2}\right]_{z=0}^{z=\sqrt{xy}} dy \, dx = \int_0^3 \int_0^{2x} \frac{xy}{2} dy \, dx = \int_0^3 \left[\frac{xy^2}{4}\right]_{y=0}^{y=2x} dx = \int_0^3 x^3 dx = \left[\frac{x^4}{4}\right]_{x=0}^{x=3} = \frac{81}{4}.$$