(6) Calculeu $\iint_R x^y \, dx\,dy$ en $R = [0,1] \times [a,b]$, essent $0 < a < b$, i deduïu el valor de la integral $\int_0^1 \frac{x^b - x^a}{\ln x} dx$

$$I = \iint_R x^y \, dx\,dy = \int_0^1 \left(\int_a^b x^y \, dy \right) dx = \int_0^1 \left(\int_a^b e^{y \ln x} \, dy \right) =$$

$$= \int_0^1 \left[\frac{e^{y \ln x}}{\ln x} \right]_{y=a}^{y=b} dx = \int_0^1 \frac{x^b - x^a}{\ln x} dx$$

$$I = \int_a^b \left(\int_0^1 x^y \, dx \right) dy = \int_a^b \left[\frac{x^{y+1}}{y+1} \right]_{x=0}^{x=1} dy = \int_a^b \frac{1}{y+1} dy =$$

$$= \left[\ln(y+1) \right]_{y=a}^{y=b} = \ln(b+1) - \ln(a+1)$$