(5) Troben les següents integrals dobles en els rectangles que s'indiquen.

(b) $I = \iint_R \frac{x^2}{1+y^2}\,dx\,dy \quad , \quad R = [0,1]\times[0,1].$

$$I = \left(\int_0^1 x^2\,dx\right)\left(\int_0^1 \frac{dy}{1+y^2}\right) = \left[\frac{x^3}{3}\right]_{x=0}^{x=1} \cdot \left[\arctan(y)\right]_{y=0}^{y=1} =$$

$$= \frac{1}{3}(\arctan(1) - \arctan(0)) = \frac{1}{3}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{12}.$$

(c) $I = \iint_R y\ln(x)\,dx\,dy \quad , \quad R = [1,e]\times[1,e]$

$$I = \left(\int_1^e \ln(x)\,dx\right)\left(\int_1^e y\,dy\right) = \left[x\ln x - x\right]_{x=1}^{x=e} \cdot \left[\frac{y^2}{2}\right]_{y=1}^{y=e} =$$

$$= \left[e\underbrace{\ln e}_{=1} - e - (1\cdot\underbrace{\ln(1)}_{=0} - 1)\right]\cdot\left[\frac{e^2}{2} - \frac{1}{2}\right] = \frac{1}{2}(e^2-1)$$

on $\int \ln(x)\,dx \underset{\uparrow}{=} x\ln x - \int x\frac{dx}{x} = x\ln x - \int dx = x\ln x - x.$

Parts $\begin{cases} u = \ln x \to du = \frac{dx}{x} \\ dv = dx \to v = x \end{cases}$

(e) $I = \iint_R \frac{1}{(x+2y)^2}\,dx\,dy \quad , \quad R = [2,5]\times[1,3]$

$$I = \int_1^3\left(\int_2^5 \frac{dx}{(x+2y)^2}\right)dy = \int_1^3\left(\int_2^5 (x+2y)^{-2}\,dx\right)dy =$$

$$= \int_1^3 \left[-(x+2y)^{-1}\right]_{x=2}^{x=5} dy = -\int_1^3\left(\frac{1}{5+2y} - \frac{1}{2+2y}\right)dy =$$

$$= -\left[\frac{\ln(5+2y)}{2} - \frac{\ln(2+2y)}{2}\right]_{y=1}^{y=3} = -\frac{1}{2}\left[\ln(11) - \ln(8) - (\ln(7) - \ln(4))\right] =$$

$$= -\frac{1}{2}\ln\left(\frac{11\times 4}{8\times 7}\right) = \frac{1}{2}\ln\left(\frac{14}{11}\right)$$

(f) $I = \iint_R e^y \sin\left(\frac{x}{y}\right) dx\,dy$, $R = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \times [1,2]$

$$I = \int_1^2 e^y \left(\int_{-\pi/2}^{\pi/2} \sin(x/y)\,dx \right) dy = \int_1^2 e^y \left[-\frac{\cos(x/y)}{1/y} \right]_{x=-\pi/2}^{x=\pi/2} dy =$$

$$= -\int_1^2 y e^y \Big(\underbrace{\cos\left(\frac{\pi}{2y}\right) - \cos\left(-\frac{\pi}{2y}\right)}_{=0} \Big) dy = 0.$$