(32) Resoleu les següents equacions sabent que admeten un factor integrant que només depèn de y.

Recordem: $P(x,y) + Q(x,y)\, y' = 0$

Calculem: $K := -\dfrac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{P}$. Si K només depèn de y, llavors $\mu(y) = e^{\int K(y)\, dy}$ factor integrant

(a) $\underbrace{2xy^4e^y + 2xy^3 + y}_{P(x,y)} + (\underbrace{x^2y^4e^y - x^2y^2 - 3x}_{Q(x,y)})\, y' = 0$

$$\left.\begin{aligned} \frac{\partial P}{\partial y} &= 8xy^3e^y + 2xy^4e^y + 6xy^2 + 1 \\ \frac{\partial Q}{\partial x} &= 2xy^4e^y - 2xy^2 - 3 \end{aligned}\right\} \text{ edo no exacta}$$

$$K = -\frac{8xy^3e^y + 8xy^2 + 4}{2xy^4e^y + 2xy^3 + y} = -4/y \quad \text{només depèn de } y$$

$$\mu(y) = e^{\int -4/y\, dy} = e^{-4\ln y} = e^{\ln y^{-4}} = y^{-4} \quad \text{factor integrant.}$$

Resolem l'equació exacta. Busquem $U(x,y)$ t.q.

$$\frac{\partial U}{\partial x} = \mu P \quad ; \quad \frac{\partial U}{\partial y} = \mu Q.$$

- $\dfrac{\partial U}{\partial x} = \mu P = 2xe^y + 2x/y + 1/y^3 \Rightarrow U(x,y) = x^2e^y + \dfrac{x^2}{y} + \dfrac{x}{y^3} + \varphi(y)$

integrem rsp. x.

- $\dfrac{\partial U}{\partial y} = x^2e^y - \dfrac{x^2}{y^2} - \dfrac{3x}{y^4} + \varphi'(y) = \mu Q = x^2e^y - \dfrac{x^2}{y^2} - \dfrac{3x}{y^4}$

cal: $\varphi'(y) = 0 \Rightarrow \varphi(y) = \text{const.}$

Així: $U(x,y) = x^2e^y + \dfrac{x^2}{y} + \dfrac{x}{y^3}$. Solució $U(x,y) = C$.