(31) Resoleu les següents equacions sabent que admeten un factor integrant que només depèn de x.

> **Recordem:** $P(x,y) + Q(x,y)\,y' = 0$
>
> Calculem: $K := \dfrac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q}$
>
> Si K només depèn de x, $K = K(x)$, llavors
>
> $\mu(x) = e^{\int K(x)\,dx}$ factor integrant

(a) $\underbrace{\frac{y^2}{2} + 2ye^x}_{= P(x,y)} + \underbrace{(y + e^x)}_{= Q(x,y)}\,y' = 0$

$\frac{\partial P}{\partial y} = y + 2e^x \neq e^x = \frac{\partial Q}{\partial x} \Rightarrow$ edo no exacta.

$K = \dfrac{(y+2e^x) - e^x}{y + e^x} = 1$ només depèn de x.

$\mu(x) = e^{\int 1\,dx} = e^x$ factor integrant.

Resolem l'equació exacta. Busquem $U(x,y)$ t.q.

$\frac{\partial U}{\partial x} = \mu P$ i $\frac{\partial U}{\partial y} = \mu Q$.

- $\frac{\partial U}{\partial y} = \mu Q = ye^x + e^{2x} \underset{\text{integrem resp. } y}{\Rightarrow} U(x,y) = \frac{y^2}{2}e^x + ye^{2x} + \varphi(x)$
- $\frac{\partial U}{\partial x} = \frac{y^2}{2}e^x + 2ye^{2x} + \varphi'(x) = \mu P = \frac{y^2}{2}e^x + 2y e^{2x}$

cal: $\varphi'(x) = 0 \Rightarrow \varphi(x) = \text{const.} \Rightarrow U(x,y) = \frac{y^2}{2}e^x + ye^{2x}$

Solució $U(x,y) = C$.

(b) $\underbrace{x+y^2}_{\overset{\shortparallel}{P(x,y)}} \underbrace{-2xyy'}_{\overset{\shortparallel}{Q(x,y)}} = 0$

$\frac{\partial P}{\partial y} = 2y \neq -2y = \frac{\partial Q}{\partial x} \Rightarrow$ edo no exacta.

$K = \frac{2y-(-2y)}{-2xy} = -\frac{2}{x}$ només depèn de x.

$\mu(x) = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2}$ factor integrant.

Resolem l'equació exacta. Busquem $U(x,y)$ t. q.

$\frac{\partial U}{\partial x} = \mu P$ i $\frac{\partial U}{\partial y} = \mu Q$

- $\frac{\partial U}{\partial y} = \mu Q = -2y/x \underset{\uparrow}{\Rightarrow} U(x,y) = -y^2/x + \varphi(x)$

integrem resp. y

- $\frac{\partial U}{\partial x} = \frac{y^2}{x^2} + \varphi'(x) = \mu Q = \frac{1}{x} + \frac{y^2}{x^2} \Rightarrow \varphi'(x) = \frac{1}{x} \Rightarrow$

$\Rightarrow \varphi(x) = \ln x + \text{Const.}$

Així: $U(x,y) = -y^2/x + \ln x$ i la solució és $U(x,y)=c$