(30) Vegem que en cada cas $\mu(x,y)$ és un factor integrant de l'equació diferencial donada i resolem l'equació diferencial exacta obtinguda.

(a) $\underbrace{-y^2}_{P(x,y)}+\underbrace{(x^2+xy)}_{Q(x,y)}y' = 0$, $\quad \mu(x,y) = \dfrac{1}{x^2y}$

$$\frac{\partial P}{\partial y} = -2y \neq 2x+y = \frac{\partial Q}{\partial x} \Rightarrow \text{edo no exacta.}$$

$$\left.\begin{aligned} \frac{\partial}{\partial y}(\mu P) &= \frac{\partial}{\partial y}\left(-\frac{y}{x^2}\right) = -\frac{1}{x^2} \\ \frac{\partial}{\partial x}(\mu Q) &= \frac{\partial}{\partial x}\left(\frac{1}{y}+\frac{1}{x}\right) = -\frac{1}{x^2} \end{aligned}\right\} \Rightarrow \mu P + \mu Q y' = 0 \text{ edo exacta.}$$

Resolem l'edo exacta. Busquem $U(x,y)$ t. q.

$$\frac{\partial U}{\partial x} = \mu P \quad i \quad \frac{\partial U}{\partial y} = \mu Q.$$

· $\dfrac{\partial U}{\partial x} = \mu P = -\dfrac{y}{x^2} \Rightarrow U(x,y) = \dfrac{y}{x} + \varphi(y)$

integrem resp. x

· $\dfrac{\partial U}{\partial y} = \dfrac{1}{x} + \varphi'(y) = \mu Q = \dfrac{1}{y} + \dfrac{1}{x} \Rightarrow \varphi'(y) = \dfrac{1}{y} \Rightarrow$

$\Rightarrow \varphi(y) = \ln|y| + \text{Const.}$

Llavors: $U(x,y) = \dfrac{y}{x} + \ln|y| + \text{Const.}$

Solució: $U(x,y) = C.$

(b) $\underbrace{(x^2+2xy-y^2)}_{P(x,y)} + \underbrace{(y^2+2xy-x^2)}_{Q(x,y)} y' = 0$, $\mu(x,y) = \dfrac{1}{(x+y)^2}$

$$\frac{\partial P}{\partial y} = 2x-2y \neq 2y-2x = \frac{\partial Q}{\partial x} \Rightarrow \text{ edo no exacta.}$$

$$\frac{\partial}{\partial y}(\mu P) = \frac{\partial}{\partial y}\left[\frac{x^2+2xy-y^2}{(x+y)^2}\right] = \frac{\partial}{\partial y}\left[\frac{(x+y)^2-2y^2}{(x+y)^2}\right] = \frac{\partial}{\partial y}\left(1-\frac{2y^2}{(x+y)^2}\right) =$$

$$= -\frac{4y}{(x+y)^2} + \frac{4y^2}{(x+y)^3} = \frac{-4xy}{(x+y)^3}$$

$$\frac{\partial}{\partial x}(\mu Q) = \frac{\partial}{\partial x}\left[\frac{y^2+2xy-x^2}{(x+y)^2}\right] = \frac{\partial}{\partial x}\left[1-\frac{2x^2}{(x+y)^2}\right] = \dots = \frac{-4xy}{(x+y)^3}$$

Per tant: $\mu P + \mu Q y' = 0$ és exacta. Resolem-la:

Busquem $U(x,y)$ t. q. $\dfrac{\partial U}{\partial x} = \mu P$ i $\dfrac{\partial U}{\partial y} = \mu Q$.

· $\dfrac{\partial U}{\partial y} = \mu Q = 1 - \dfrac{2x^2}{(x+y)^2} \Rightarrow U(x,y) = y + \dfrac{2x^2}{(x+y)} + \varphi(x)$

integrem resp. y

· $\dfrac{\partial U}{\partial x} = \dfrac{4x(x+y)-2x^2}{(x+y)^2} + \varphi'(x) = \dfrac{2x^2+4xy}{(x+y)^2} + \varphi'(x) = \mu P = \dfrac{x^2+2xy-y^2}{(x+y)^2}$

Per tant: $\varphi'(x) = \dfrac{-x^2-2xy-y^2}{(x+y)^2} = -1 \Rightarrow \varphi(x) = -x + \text{Const.}$

Per tant: $U(x,y) = y - x + \dfrac{2x^2}{x+y} + \text{Const.} = \dfrac{y^2+x^2}{x+y}$