(29) Mitjançant l'us del desenvolupament de Taylor de funcions d'una variable (coneguts a priori), calcular els desenvolupaments de Taylor en l'origen fins a termes de grau 2 inclosos de les següents funcions.

(a) $f(x,y) = e^{xy} \ln(1+x+y)$

$$e^z = 1 + z + z^2/2! + O_3(z) \Rightarrow e^{xy} = 1 + xy + O_4(x,y)$$

$$\ln(1+z) = z - z^2/2 + O_3(z) \Rightarrow \ln(1+x+y) = x+y - \frac{(x+y)^2}{2} + O_3(x,y)$$

$$f(x,y) = [1 + xy + O_4(x,y)] \cdot [x+y-(x+y)^2/2 + O_3(x,y)] =$$

$$= x + y - \frac{x^2}{2} - xy - y^2/2 + O_3(x,y).$$

(c) $f(x,y) = \dfrac{1}{1+x+y}$

$$\frac{1}{1-z} = 1 + z + z^2 + O_3(z)$$

$$f(x,y) = \frac{1}{1-(-x-y)} = 1 + (-x-y) + (-x-y)^2 + O_3(x,y) =$$

$$= 1 - x - y + x^2 + 2xy + y^2 + O_3(x,y).$$

(b) $f(x,y) = e^x \cos y$

$$f(x,y) = [1 + x + x^2/2! + O_3(x)] \cdot [1 - y^2/2! + O_3(y)] =$$

$$= 1 + x + \frac{x^2}{2} - \frac{y^2}{2} + O_3(x,y)$$

cd) $f(x,y,z) = e^{x+y}\sqrt{1+x}\,\cos(x+y+z)$

$$\sqrt{1+x} = (1+x)^{1/2} = 1 + \frac{1}{2}x + \frac{1/2(1/2-1)}{2}x^2 + O_3(x) =$$

$$= 1 + 1/2\, x - \frac{1}{8}x^2 + O_3(x).$$

$$f(x,y,z) = \left[1 + x + y + \frac{(x+y)^2}{2} + O_3(x,y)\right]\cdot\left[1 + \frac{x}{2} - \frac{x^2}{8} + O_3(x)\right]\cdot$$

$$\cdot\left[1 - (x+y+z)^2/2! + O_4(x,y,z)\right] =$$

$$= 1 + \left\{\frac{x}{2} + x + y\right\} +$$

$$+ \left\{\frac{(x+y)^2}{2} - \frac{1}{8}x^2 - \frac{(x+y+z)^2}{2} + (x+y)\frac{x}{2}\right\} + O_3(x,y,z)$$

$$= 1 + \frac{3}{2}x + y + \left\{\frac{x^2}{2} + xy + \frac{y^2}{2} - \frac{1}{8}x^2 -\right.$$

$$\left. - \frac{x^2}{2} - \frac{y^2}{2} - \frac{z^2}{2} - xy - xz - yz + \frac{x^2}{2} + \frac{xy}{2}\right\} + O_3(x,y,z)$$

$$= 1 + \frac{3}{2}x + y + \frac{3}{8}x^2 - \frac{z^2}{2} + \frac{xy}{2} - xz - yz + O_3(x,y,z)$$