(16) Sigui $f(u,v,w) = (e^{u-w}, \ln(u+v) + \sin(u+v-w))$ i $g(x,y) = (e^x, \sin(y-x), e^{-y})$. Calculeu $D(f \circ g)(0,0)$ mitjançant la regla de la cadena.

$$g(0,0) = (e^0, \sin(0-0), e^{-0}) = (1,0,1)$$

$$\frac{\partial g}{\partial x} = (e^x, \cos(y-x)\cdot(-1), 0)$$

$$\frac{\partial g}{\partial y} = (0, \cos(y-x), -e^{-y})$$

$$Dg(x,y) = \begin{bmatrix} e^x & 0 \\ -\cos(y-x) & \cos(y-x) \\ 0 & -e^{-y} \end{bmatrix}, \quad Dg(0,0) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\frac{\partial f}{\partial u} = \left(e^{u-w}, \frac{1}{u+v} + \cos(u+v-w)\right) \qquad \frac{\partial f}{\partial u}(1,0,1) = (1, 1+\cos(0)) = (1,2)$$

$$\frac{\partial f}{\partial v} = \left(0, \frac{1}{u+v} + \cos(u+v-w)\right) \qquad \frac{\partial f}{\partial v}(1,0,1) = (0, 1+\cos(0)) = (0,2)$$

$$\frac{\partial f}{\partial w} = (-e^{u-w}, -\cos(u+v-w)) \qquad \frac{\partial f}{\partial w}(1,0,1) = (-1, -\cos(0)) = (-1,-1)$$

$$D(f\circ g)(0,0) = Df(g(0,0))\cdot Dg(0,0) = Df(1,0,1)\cdot Dg(0,0) =$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}.$$