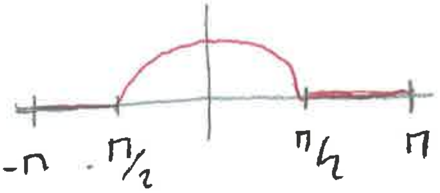
(33) Troben la Sèrie de Fourier de $f(x)=\begin{cases} 0, & si\ \frac{\pi}{2}\le x\le \pi \\ \cos x, & si\ |x|<\frac{\pi}{2}\end{cases}$



$$f(x) \sim \mathcal{F}[f](x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{T}x\right) + b_n \sin\left(\frac{n\pi}{T}x\right)\right) =$$

$$= a_0/2 + \sum_{n=1}^{\infty} a_n \cos(nx)$$

($T=\pi$; f parell $\Rightarrow b_n=0$)

$$a_0 = \frac{1}{T}\int_{-T}^{T} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\left\{\int_0^{\pi/2}\cos x\,dx + \int_{\pi/2}^{\pi} 0\cdot dx\right\} =$$

$$= \frac{2}{\pi}\left[\sin x\right]_{x=0}^{x=\pi/2} = \frac{2}{\pi}$$

$$a_n = \frac{1}{T}\int_{-T}^{T} f(t)\cos\left(\frac{n\pi}{T}x\right)dx = \frac{2}{\pi}\int_0^{\pi/2}\cos(x)\cdot\cos(nx)\,dx =$$

$$= \frac{1}{\pi}\int_0^{\pi/2}\left\{\cos(n-1)x + \cos(n+1)x\right\}dx =$$

$\cos\alpha\cdot\cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)$

$$= \frac{1}{\pi}\left[\frac{\sin(n-1)x}{n-1} + \frac{\sin(n+1)x}{n+1}\right]_{x=0}^{x=\pi/2} = \frac{1}{\pi}\left(\frac{\sin(n-1)\pi/2}{n-1} + \frac{\sin(n+1)\pi/2}{n+1}\right) =$$

$(n\neq 1)$

$= \begin{cases} = 0, \ si\ n=2k+1 \text{ (senar)}, \ k\geq 1 \\ (n=2k-1) \end{cases}$

$(n=2k)$

$$= \frac{1}{\pi}\left(\frac{\sin(2k-1)\pi/2}{2k-1} + \frac{\sin(2k+1)\pi/2}{2k+1}\right) = \frac{1}{\pi}\left(\frac{(-1)^{k+1}}{2k-1} + \frac{(-1)^k}{2k+1}\right) = \frac{(-1)^k}{\pi}\left(\frac{-1}{2k-1}+\frac{1}{2k+1}\right) =$$

$$= \frac{(-1)^k}{\pi}\,\frac{-(2k+1)+(2k-1)}{4k^2-1} = \frac{(-1)^k}{\pi}\,\frac{(-2)}{4k^2-1} = \frac{2(-1)^{k+1}}{\pi(4k^2-1)}, \quad si\ n=2k \text{ (parell)}, \ k\geq 1$$

Fem a part el cas $n=1$: $a_1 = \frac{1}{\pi}\int_0^{\pi/2}(1+\cos(2x))\,dx = \dots = \frac{1}{2}$

$$f(x) \sim \frac{1}{\pi} + \frac{1}{2}\cos x + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{4k^2-1}\cos(2kx).$$