(31) Troben la sèrie de Fourier de $f(x) = \begin{cases} 0, & \text{si } -\pi < x < 0 \\ x, & \text{si } 0 \leq x < \pi \end{cases}$ a l'interval $-\pi < x < \pi$.

Quant val $\mathcal{F}[f]$ a $x = \frac{7\pi}{2}$? I quant val a $x = 401\pi$?

$$\mathcal{F}[f](x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{T}x\right) + b_n \sin\left(\frac{n\pi}{T}x\right)\right) =$$

$$\underset{(T=\pi)}{=} \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

$$a_0 = \frac{1}{T}\int_{-T}^{T} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} x\,dx = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{x=0}^{x=\pi} = \frac{\pi}{2}.$$

$$a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos\left(\frac{n\pi}{T}x\right)dx = \frac{1}{\pi}\int_0^{\pi} x\cos(nx)\,dx =$$

$$= \left\{\begin{array}{l} u = x \to du = dx \\ dv = \cos(nx)\,dx \to v = \frac{\sin(nx)}{n} \end{array}\right\} = \frac{1}{\pi}\left\{\underbrace{\left[\frac{x\sin(nx)}{n}\right]_{x=0}^{x=\pi}}_{=0} - \frac{1}{n}\int_0^{\pi}\sin(nx)\,dx\right\} =$$

$$= -\frac{1}{n\pi}\left[-\frac{\cos(nx)}{n}\right]_{x=0}^{x=\pi} = \frac{1}{\pi n^2}(\cos(n\pi) - 1) = \frac{(-1)^n - 1}{\pi n^2}$$

$$b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\left(\frac{n\pi x}{T}\right)dx = \frac{1}{\pi}\int_0^{\pi} x\sin(nx)\,dx =$$

$$= \left\{\begin{array}{l} u = x \to du = dx \\ dv = \sin(nx)\,dx \to v = -\frac{\cos(nx)}{n} \end{array}\right\} = \frac{1}{\pi}\left\{\left[-\frac{x\cos(nx)}{n}\right]_{x=0}^{x=\pi} + \frac{1}{n}\int_0^{\pi}\cos(nx)\,dx\right\}$$

$$= \frac{1}{\pi}\left\{-\frac{\pi\cos(n\pi)}{n} + \underbrace{\frac{1}{n}\left[\frac{\sin(nx)}{n}\right]_{x=0}^{x=\pi}}_{=0}\right\} = \frac{(-1)^{n+1}}{n}$$

$$\mathcal{F}[f]\left(\frac{7\pi}{2}\right) = \mathcal{F}[f]\left(\frac{7\pi}{2} - 4\pi\right) = \mathcal{F}[f]\left(-\frac{\pi}{2}\right) = 0$$ f contínua en $x = -\frac{\pi}{2}$

$$\mathcal{F}[f](401\pi) = \mathcal{F}[f](401\pi - 400\pi) = \mathcal{F}[f](\pi) = \frac{\pi + 0}{2} = \pi/2$$ promig del salt en $x = \pi$.