(7) Determinar el límit, cas d'existir, de les successions següents:

(a) $a_n = \dfrac{2n}{\sqrt{n^2+1}} = \dfrac{2}{\sqrt{1+1/n^2}} \underset{n\to+\infty}{\longrightarrow} 2$

(b) $a_n = \cos\left(\frac{2}{n}\right) \underset{n\to+\infty}{\longrightarrow} \cos(0) = 1.$

(c) $a_n = \dfrac{n^p}{e^n}$ $(p>0)$

$$\lim_{n\to+\infty} a_n \underset{\uparrow}{=} \lim_{x\to+\infty} \frac{x^p}{e^x} = \frac{\infty}{\infty} \underset{\uparrow}{=} \lim_{x\to+\infty} \frac{p\,x^{p-1}}{e^x} \underset{\uparrow}{=} \cdots \underset{\uparrow}{=} \lim_{x\to+\infty} \frac{p(p-1)\cdots 1\cdot x^0}{e^x} =$$

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$$= \lim_{x\to+\infty} \frac{p!}{e^x} = 0.$$

(d) $a_n = \dfrac{1\cdot 3\cdot 5\cdots(2n-1)}{(2n)^n}$

observem : $a_1 = \frac{1}{2}$; $a_2 = \frac{1\cdot 3}{4^2} = \frac{1}{4}\cdot\frac{3}{4}$; $a_3 = \frac{1\cdot 3\cdot 5}{6^3} = \frac{1}{6}\cdot\frac{3}{6}\cdot\frac{5}{6}$

Així, $a_n = \frac{1}{2n}\cdot\frac{3}{2n}\cdot\frac{5}{2n}\cdots\frac{2n-1}{2n}$, verifica les desigualtats:

$0 \leq a_n \leq \frac{1}{2n}$, ja que $\frac{3}{2n}<1$, $\frac{5}{2n}<1, \ldots, \frac{2n-1}{2n}<1$.

Per tant, aplicant el teorema de l'encaix per successions tenim:

$$\lim_{n\to+\infty} 0 = \lim_{n\to+\infty} \frac{1}{2n} = 0 \Rightarrow \lim_{n\to+\infty} a_n = 0.$$

(e) $a_n = \dfrac{\ln(n^3)}{2n}$

$$\lim_{n\to+\infty} a_n \underset{\uparrow}{=} \lim_{x\to+\infty} \frac{\ln(x^3)}{2x} = \frac{3}{2}\lim_{x\to+\infty}\frac{\ln(x)}{x} = \frac{\infty}{\infty} \underset{\uparrow}{=} \frac{3}{2}\lim_{x\to+\infty}\frac{1/x}{1} = 0.$$

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(f) $a_n = \dfrac{\cos(\pi n)}{n^2} = \dfrac{(-1)^n}{n^2}$ [observem: $(-1)^n = 1$, si n parell; $(-1)^n = -1$ si n senar]

$$\lim_{n\to+\infty} \frac{(-1)^n}{n^2} = 0.$$