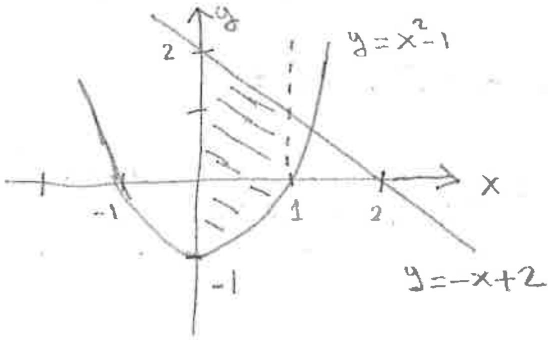
(25) Troben l'àrea delimitada per les funcions següents:

(a) $y = x^2-1$, $y = -x+2$, $x=0$, $x=1$



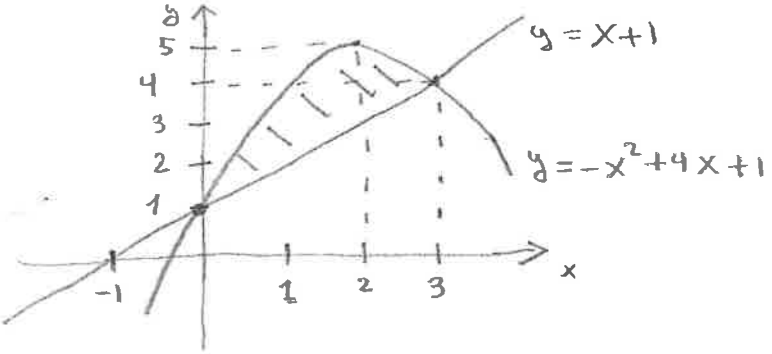
$$\text{Àrea} = \int_0^1 [(-x+2)-(x^2-1)]\,dx = \int_0^1 (3-x-x^2)\,dx =$$

$$= \left[3x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{x=0}^{x=1} = 3-\frac{1}{2}-\frac{1}{3} = \frac{13}{6}$$

(b) $f(x) = -x^2+4x+1$, $g(x) = x+1$

Primer busquem els punts de tall de les gràfiques de $f(x)$ i $g(x)$:

$$-x^2+4x+1 = x+1 \iff x^2-3x=0 \Rightarrow x(x-3)=0 \Rightarrow x=0 \text{ ó } x=3$$



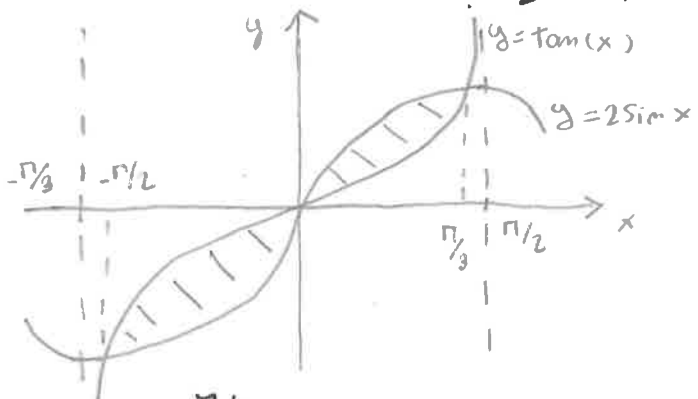
← la paràbola té el seu vèrtex en $(x,y)=(2,5)$, on $y'=0$.

$$\text{Àrea} = \int_0^3 [(-x^2+4x+1)-(x+1)]\,dx = \int_0^3 (-x^2+3x)\,dx = \left[-\frac{x^3}{3}+\frac{3x^2}{2}\right]_{x=0}^{x=3} =$$

$$= -\frac{3^3}{3}+\frac{3\cdot 3^2}{2} = \frac{9}{2}(-2+3) = 9/2.$$

(c) $f(x) = 2\sin x$, $g(x) = \tan(x)$, $-\pi/3 \le x \le \pi/3$

observem: $f(\pi/3) = 2\sin(\pi/3) = 2\frac{\sqrt{3}}{2} = \sqrt{3}$; $g(\pi/3) = \dfrac{\sin(\pi/3)}{\cos(\pi/3)} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$



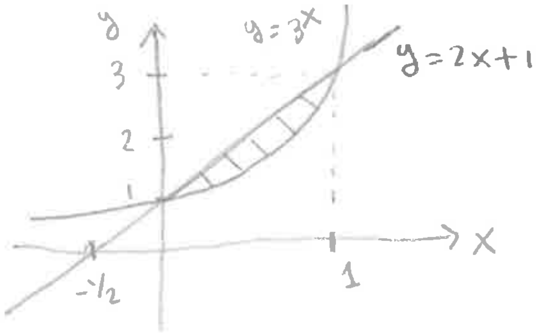
Per simetria ($f(x)$ i $g(x)$ són funcions senars) l'àrea dels 2 lòbuls ($x\in[0,\pi/3]$ i $x\in[-\pi/3,0]$) és la mateixa.

$$\text{Àrea} = 2\int_0^{\pi/3} [2\sin x - \tan(x)]\,dx = 2\left[-2\cos x + \ln(\cos x)\right]_{x=0}^{x=\pi/3} =$$

$$= 2\left[-2\cos(\pi/3)+\ln(\cos(\pi/3))+2\cos(0)-\ln(\cos(0))\right] = 2(1-\ln 2)$$

(d) $f(x) = 3^x$, $g(x) = 2x+1$

observem que $f(x) = g(x) \Leftrightarrow 3^x = 2x+1$ té solucions (òbvies) $x = 0, 1$.



$$\text{Àrea} = \int_0^1 (2x+1-3^x)\,dx =$$

$$= \int_0^1 (2x+1-e^{x\cdot\ln 3})\,dx = \left[x^2+x-\frac{e^{x\cdot \ln 3}}{\ln 3}\right]_{x=0}^{x=1} =$$

$$= 1^2+1-\frac{3^1}{\ln 3} - \left(0^2+0-\frac{3^0}{\ln 3}\right) = 2 - \frac{2}{\ln 3}.$$