(22) Analitzeu si les següents integrals impròpies són convergents i, en aquest cas, calculeu el seu valor.

(a) $\int_1^{+\infty} \frac{4}{x^2+1}\,dx = \lim_{M\to+\infty} \int_1^M \frac{4}{x^2+1}\,dx = \lim_{M\to+\infty} 4\arctan(x)\Big|_{x=1}^{x=M} =$

$= \lim_{M\to+\infty} 4(\arctan(M)-\arctan(1)) = 4\left(\frac{\pi}{2}-\frac{\pi}{4}\right) = 4\cdot\frac{\pi}{4} = \pi.$

(b) $\int_0^{+\infty} e^{-x}\cos x\,dx$

$\int e^{-x}\cos x\,dx = \left\{\begin{array}{l} u=\cos x \to du=-\sin x\,dx \\ dv = e^{-x}dx \to v=-e^{-x}\end{array}\right\} = -e^{-x}\cos x - \int e^{-x}\sin x\,dx =$

$= \left\{\begin{array}{l} u=\sin x \to du=\cos x\,dx \\ dv = e^{-x}dx \to v=-e^{-x}\end{array}\right\} = -e^{-x}\cos x - \left[-e^{-x}\sin x + \int e^{-x}\cos x\,dx\right]$

Així: $\int e^{-x}\cos x\,dx = -\frac{1}{2}e^{-x}\cos x + \frac{1}{2}e^{-x}\sin x$

$\int_0^{+\infty} e^{-x}\cos x\,dx = \lim_{M\to+\infty}\int_0^M e^{-x}\cos x\,dx = \lim_{M\to+\infty}\left[-\frac{1}{2}e^{-x}\cos x + \frac{1}{2}e^{-x}\sin x\right]_{x=0}^{x=M} =$

$= \lim_{M\to+\infty}\left(-\frac{1}{2}e^{-M}\cos M + \frac{1}{2}e^{-M}\sin M + \frac{1}{2}\right) = \frac{1}{2}.$

(c) $\int_0^8 \frac{dx}{\sqrt[3]{8-x}} = \lim_{M\to 8^-}\int_0^M (8-x)^{-1/3}dx = \lim_{x\to 8^-} \frac{(8-x)^{2/3}}{2/3\,(-1)}\Big|_{x=0}^{x=M} =$

$= \lim_{x\to 8^-} \frac{3}{2}\left(8^{2/3} - (8-M)^{2/3}\right) = 6.$

(d) $\int_0^{\pi/2} \tan\theta\,d\theta = \lim_{M\to\frac{\pi}{2}^-}\int_0^M \tan\theta\,d\theta = \lim_{M\to\frac{\pi}{2}^-}\left[-\ln|\cos\theta|\right]_{\theta=0}^{\theta=M} =$

$= \lim_{M\to\frac{\pi}{2}^-} -\ln|\cos M| = +\infty$ divergent!