(12) (T$^a$ del valor mig per a integrals) Si $f:[a,b]\to\mathbb{R}$ contínua, $\exists\, c\in(a,b)$ t.q. $\int_a^b f(x)\,dx = f(c)\cdot(b-a)$.

Determinem $c$ per a les integrals següents:

(a) $I=\int_0^3 x^3\,dx$ , $f(x)=x^3$, $a=0$, $b=3$

$$\left.\begin{array}{l} I=\left[x^4/4\right]_{x=0}^{x=3} = 3^4/4 - 0^4/4 = 81/4 \\ \parallel \\ f(c)\cdot(b-a) = c^3\cdot 3 \end{array}\right\} \Rightarrow c^3 = 27/4 \Rightarrow c = \frac{3}{\sqrt[3]{4}}$$

(b) $I=\int_0^2 (x-2\sqrt{x})\,dx$, $f(x)=x-2\sqrt{x}$, $a=0$, $b=2$

$$\left.\begin{array}{l} I=\left[\frac{x^2}{2} - 2\frac{x^{3/2}}{3/2}\right]_{x=0}^{x=2} = \frac{2^2}{2} - \frac{4}{3}\cdot 2^{3/2} - 0 \underset{\underset{3/2=1+1/2}{\uparrow}}{=} 2-\frac{8}{3}\sqrt{2} \\ \parallel \\ f(c)\cdot(b-a) = (c-2\sqrt{c})\cdot 2 \end{array}\right\}$$

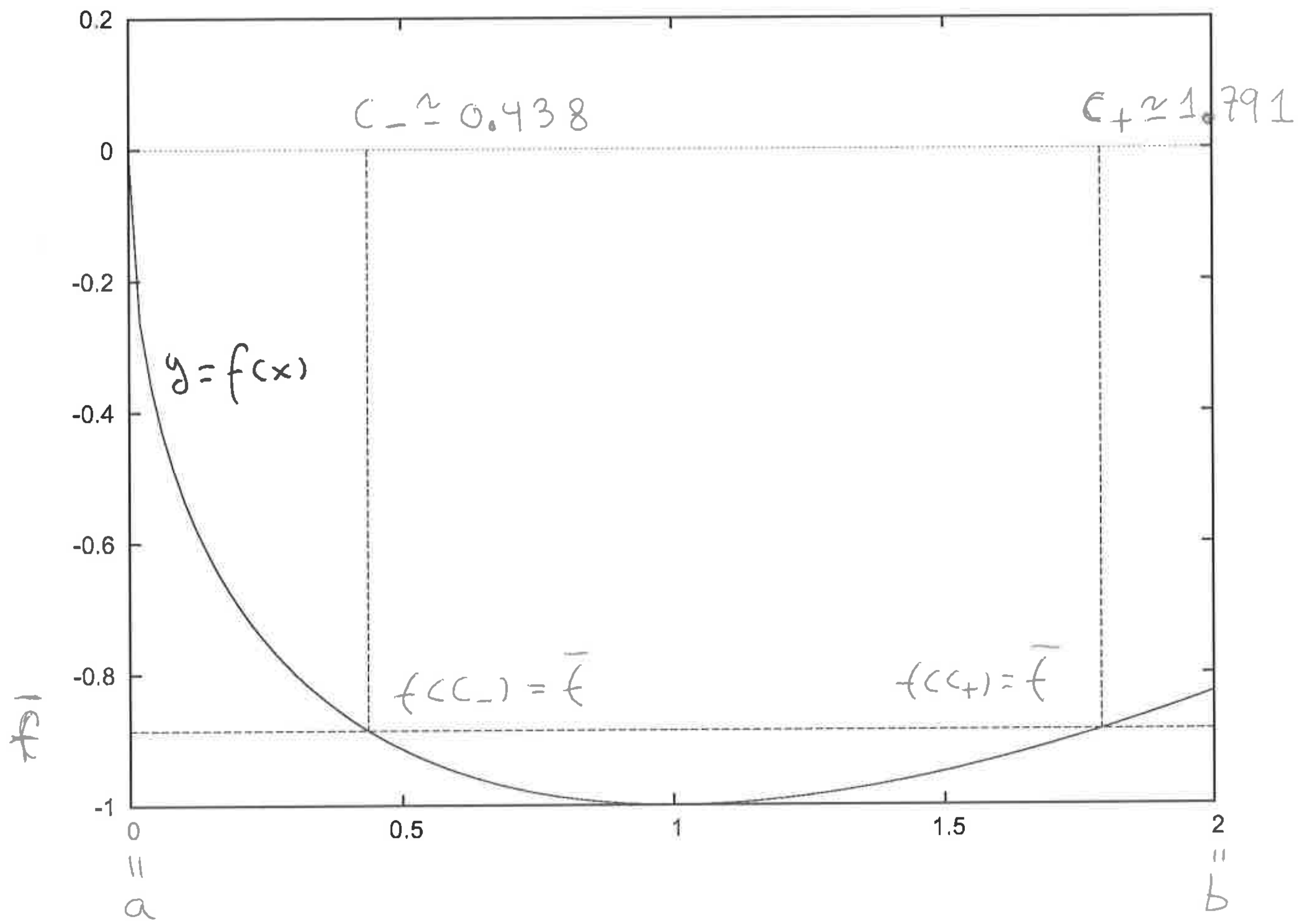
obtenim l'equació: $2(c-2\sqrt{c}) = 2-8/3\sqrt{2}$ ,

on si anomenem $z=\sqrt{c}$ , llavors $z^2-2z+4/3\sqrt{2}-1=0$

$$\underset{\underset{\sqrt{c}}{\parallel}}{z} = \frac{2\pm\sqrt{4-4(4/3\sqrt{2}-1)}}{2} = 1\pm\sqrt{1-4\sqrt{2}/3+1} = 1\pm\sqrt{\underbrace{2-4\sqrt{2}/3}_{>0}}$$

obtenim: $c_\pm = \left[1\pm\sqrt{2-4\sqrt{2}/3}\right]^2$ dues solucions que pertanyen totes dues a l'interval $[a,b]=[0,2]$.

Ho podem veure fàcilment si dibuixem la gràfica de la funció vs. el seu valor promig.




$$f(x) = x - 2\sqrt{x}$$

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2}\int_0^2 (x - 2\sqrt{x})\,dx = 1 - \frac{4}{3}\sqrt{2} \simeq -0.885618$$