4 Utilitzeu la integració per parts per provar les fórmules següents:

(a) $\int x^n \sin x\, dx = -x^n \cos x + n\int x^{n-1}\cos x\, dx$.

$$\int x^n \sin x\, dx = \left\{\begin{array}{l} u = x^n \to du = n x^{n-1} dx \\ dv = \sin x\, dx \to v = -\cos x \end{array}\right\} \equiv (a).$$

(b) $\int x^n \cos x\, dx = x^n \sin x - n\int x^{n-1} \sin x\, dx$

Ídem: $u = x^n \to du = n x^{n-1} dx$; $dv = \cos x\, dx \to v = \sin x$

(c) $\int x^n \ln x\, dx = \dfrac{x^{n+1}}{(n+1)^2}(-1 + (n+1)\ln x) + C$.

$$\int x^n \ln x\, dx = \left\{\begin{array}{l} u = \ln x \to du = dx/x \\ dv = x^n dx \to v = \dfrac{x^{n+1}}{n+1} \end{array}\right\} = \frac{x^{n+1}}{n+1}\ln x - \int \frac{x^n}{n+1} dx \equiv (c).$$

(d) $\int x^n e^{ax} dx = \dfrac{x^n e^{ax}}{a} - \dfrac{n}{a}\int x^{n-1} e^{ax} dx$

$$\int x^n e^{ax} dx = \left\{\begin{array}{l} u = x^n \to du = n x^{n-1} \\ dv = e^{ax} dx \to v = \dfrac{e^{ax}}{a} \end{array}\right\} \equiv (d)$$

(e) $\int e^{ax} \sin bx\, dx = \dfrac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2} + C$

$$I = \int e^{ax} \sin bx\, dx = \left\{\begin{array}{l} u = e^{ax} \to du = a e^{ax} dx \\ dv = \sin bx\, dx \to v = -\dfrac{\cos bx}{b} \end{array}\right\} = -\frac{1}{b}e^{ax}\cos bx + \frac{a}{b}\int e^{ax}\cos bx\, dx =$$

$$= \left\{\begin{array}{l} u = e^{ax} \to du = a e^{ax} dx \\ dv = \cos bx\, dx \to v = \dfrac{\sin bx}{b} \end{array}\right\} = -\frac{1}{b}e^{ax}\cos bx + \frac{a}{b}\left[\frac{e^{ax}\sin bx}{b} - \frac{a}{b}\int e^{ax}\sin bx\, dx\right]$$

d'on: $I = -\dfrac{1}{b}e^{ax}\cos bx + \dfrac{a}{b^2}e^{ax}\sin bx - \dfrac{a^2}{b^2} I$. Aïllant I, surt (e).

(f) $\int e^{ax}\cos bx\, dx = \dfrac{e^{ax}(a\cos bx + b\sin bx)}{a^2+b^2} + C$

Fem $J = \int e^{ax}\cos bx\, dx$ i a (e) hem vist: $I = -\dfrac{1}{b}e^{ax}\cos bx + \dfrac{a}{b} J$

d'aquí obtenim J a partir de I.

Apliquem-les per calcular les primitives:

$$\int x^5 \ln x \, dx \underset{\substack{\uparrow \\ (c)\ n=5}}{=} \frac{x^6}{6^2}(-1+6\ln x)+C$$

$$\int x^3 e^{2x} dx \underset{\substack{\uparrow \\ (d),\ n=3,\ a=2}}{=} \frac{x^3 e^x}{2} - \frac{3}{2}\int x^2 e^{2x} dx \underset{\substack{\uparrow \\ (d)\ n=2,\ a=2}}{=} \frac{x^3 e^x}{2} - \frac{3}{2}\left[\frac{x^2 e^{2x}}{2} - \frac{2}{2}\int x e^{2x} dx\right] =$$

$$\underset{\substack{\uparrow \\ (d)\ n=1,\ a=2}}{=} \frac{x^3 e^x}{2} - \frac{3}{4}x^2 e^{2x} + \frac{3}{2}\left[\frac{x e^{2x}}{2} - \frac{1}{2}\int e^{2x} dx\right] = \frac{x^3 e^x}{2} - \frac{3}{4}x^2 e^{2x} + \frac{3}{4}x e^{2x} -$$

$$- \frac{3}{8}e^{2x} + C.$$