11 En els casos següents calculeu el límit

(a) $\lim_{x\to 0^-} \left(1+\frac{1}{x}\right) = 1-\infty = -\infty.$

(b) $\lim_{x\to 0^-} \left(x^2 - \frac{1}{x}\right) = 0+\infty = +\infty.$

(c) $\lim_{x\to \pi} \frac{\sqrt{x}}{\operatorname{cosec} x} = \lim_{x\to\pi} \sqrt{x}\cdot \sin x = 0.$

(d) $\lim_{x\to 0} \frac{x+2}{\cot x} = \lim_{x\to 0} (x+2)\operatorname{tg} x = 0.$

(e) $\lim_{x\to 1/2} x\sec(\pi x) = \lim_{x\to 1/2} \frac{x}{\cos(\pi x)} = \left\{\begin{array}{l} \lim_{x\to 1/2^+} \frac{x}{\cos(\pi x)} = -\infty \\ \lim_{x\to 1/2^-} \frac{x}{\cos(\pi x)} = +\infty \end{array}\right\} = \nexists$

(f) $\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1} = \lim_{x\to 1} \frac{(\sqrt[3]{x}-1)}{(\sqrt{x}-1)} \frac{((\sqrt[3]{x})^2+\sqrt[3]{x}+1)}{((\sqrt[3]{x})^2+\sqrt[3]{x}+1)} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{x-1}{x-1} \cdot \frac{\sqrt{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} =$

$= \lim_{x\to 1} \frac{\sqrt{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} = \frac{2}{3}$ on usem $(A-1)(A^2+A+1) = A^3-1$

$(A-1)(A+1) = A^2-1$

(g) $\lim_{x\to 0} \frac{\sqrt{1+x+x^2}-1}{x} = \lim_{x\to 0} \frac{(\sqrt{1+x+x^2}-1)(\sqrt{1+x+x^2}+1)}{x\,(\sqrt{1+x+x^2}+1)} =$

$= \lim_{x\to 0} \frac{(1+x+x^2)-1}{x(\sqrt{1+x+x^2}+1)} = \lim_{x\to 0} \frac{1+x}{\sqrt{1+x+x^2}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

(h) $\lim_{x\to a} \frac{\sqrt[m]{x}-\sqrt[m]{a}}{x-a} \underset{(a>0)}{=} \lim_{x\to a} \frac{(\sqrt[m]{x}-\sqrt[m]{a})\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}\sqrt[m]{a}+\dots+(\sqrt[m]{a})^{m-1}\right]}{(x-a)\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}\sqrt[m]{a}+\dots+(\sqrt[m]{a})^{m-1}\right]} =$

$= \lim_{x\to a} \frac{1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}\sqrt[m]{a}+\dots+(\sqrt[m]{a})^{m-1}} = \frac{1}{m(\sqrt[m]{a})^{m-1}} = \frac{1}{m\,a^{(m-1)/m}}$

(i) $\lim_{x\to 0} \frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q} \underset{(p,q>0)}{=} \lim_{x\to 0} \frac{(\sqrt{x^2+p^2}-p)}{(\sqrt{x^2+q^2}-q)} \frac{(\sqrt{x^2+p^2}+p)}{(\sqrt{x^2+p^2}+p)} \frac{(\sqrt{x^2+q^2}+q)}{(\sqrt{x^2+q^2}+q)} =$

$= \lim_{x\to 0} \frac{\sqrt{x^2+q^2}+q}{\sqrt{x^2+p^2}+p} = \frac{q+q}{p+p} = \frac{q}{p}.$